Regular Article – Theoretical Physics

# The Gribov–Zwanziger action in the presence of the gauge invariant, nonlocal mass operator Tr $\int \mathrm{d}^4x F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ in the Landau gauge

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**Abstract.** We prove that the nonlocal gauge invariant mass dimension 2 operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ can be consistently added to the Gribov–Zwanziger action, which implements the restriction of the path integral's domain of integration to the first Gribov region when the Landau gauge is considered. We identify a local polynomial action and prove the renormalizability to all orders of perturbation theory by employing the algebraic renormalization formalism. Furthermore, we also pay attention to the breaking of the BRST invariance, and to the consequences that this has for the Slavnov–Taylor identity.

**PACS.** 11.15.-q; 11.15.Tk

## 1 Introduction

It is a well known fact that $\mathrm{SU}(N)$ Yang–Mills gauge theories, described by the Euclidean action

$$S_{\mathrm{YM}} = \frac{1}{4} \int \mathrm{d}^4x \, F_{\mu\nu}^a F_{\mu\nu}^a \,, \qquad (1)$$

with $A_\mu$ the gauge potential and

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \,, \qquad (2)$$

the field strength, whereby $D_\mu^{ab}$ is the covariant derivative in the adjoint representation, given by

$$D_\mu^{ab} = \partial_\mu \delta^{ab} - g f^{abc} A_\mu^c \,, \qquad (3)$$

are asymptotically free at very high energies [1–4]. The coupling constant is sufficiently small to allow for a perturbative description, with asymptotic degrees of freedom given by massless gauge bosons. We shall not consider

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fermion matter in this paper; however, the same conclusion holds for quantum chromodynamics (QCD), written in terms of gluons and quarks. When we pass to lower energies, the coupling constant $g^2$ begins to grow, and perturbation theory starts to lose its validity. At still lower energies, the situation becomes dramatically different, as perturbation theory now completely fails, and confinement sets in, meaning that the elementary field excitations are no longer physical observables, but become confined into colorless states. The hadrons constitute the physical states of QCD.

A satisfactory understanding of the behavior of Yang–Mills theories in the low energy regime is yet unavailable. Due to the large coupling constant, nonperturbative effects have to be taken into account. The introduction of condensates, which are the (integrated) vacuum expectation value of certain local operators, allows one to parametrize certain nonperturbative effects arising from the infrared sector of e.g. the theory described by (1). Condensates give rise to power corrections, a phenomenon that can be handled using the operator product expansion. Clearly, these power corrections correspond to nonperturbative information in addition to the perturbatively calculable results. If one wants to consider the possible effects of condensates on physical quantities in a gauge theory, only gauge invariant operators are relevant. The most famous example is the dimension 4 gluon condensate $\langle \alpha_s F_{\mu\nu}^2 \rangle$, giving rise to $\frac{1}{q^4}$ power corrections in QCD. The SVZ (Shifman–

Vainshtein–Zakharov) sum rules [5] can then be used to relate the condensates to observables, and hence one may obtain certain phenomenological estimates for e.g. $\langle \alpha_s F_{\mu\nu}^2 \rangle$. This approach allows for a study of at least some aspects of QCD in an energy regime in between the confined and perturbative zone. One can still use perturbation theory using the quarks and gluons as effective degrees of freedom due to the lack of explicit knowledge of the correct physical degrees of freedom, but the results get adapted by nonperturbatively generated condensates.

In this paper, we shall introduce an action that can serve as a starting point for investigating some nonperturbative effects in gauge theories. These nonperturbative effects arise from two premises: the Gribov problem in fixing the gauge freedom, and the possibility of dynamical mass generation.

First of all, we shall be concerned about fixing the gauge. If we want to perform any kind of calculations, we must reduce the enormous gauge freedom of (1), encoded in the local gauge symmetry generated by

$$\delta_\omega A_\mu^a = -D_\mu^{ab}\omega^b, \quad \text{with} \quad \omega^b \quad \text{arbitrary}, \qquad (4)$$

to a global one by a suitable gauge fixing condition, say $\mathcal{F}(A_\mu) = 0$. In principle, by imposing a gauge condition one should select a single representative $A_\mu^*$ from the gauge orbit $A_\mu^U$, where $U$ is a generic SU($N$) gauge transformation. Unfortunately, it was shown that it is impossible to uniquely fix the gauge, a problem related to the complicated topology of the space of gauge orbits [6].

Seminal work on the existence of gauge copies was done three decades ago by Gribov in [7]. This paper is not the place to give a complete overview of the ambiguities arising when a gauge fixing is performed, we therefore kindly refer to Gribov's original paper or to the available literature, such as [8], which contains many examples and references. In particular, in [7], it was pointed out that, in the Landau and Coulomb gauges, the existence of zero modes in the Faddeev–Popov operator gives rise to gauge copies. Using the Landau gauge,

$$\partial_\mu A_\mu = 0, \qquad (5)$$

one finds that a gauge equivalent configuration $A_\mu'$, connected to $A_\mu$ via (4), also obeys $\partial_\mu A_\mu' = 0$, when

$$\mathcal{M}^{ab}\omega^b = 0, \qquad (6)$$

where $\mathcal{M}^{ab}$ denotes the Faddeev–Popov operator

$$\mathcal{M}^{ab} = -\partial_\mu \left( \partial_\mu \delta^{ab} - gf^{abc}A_\mu^c \right). \qquad (7)$$

The existence of the Gribov copies implies that the domain of integration in the path integral has to be further restricted in a suitable way. Following Gribov, it seems logical to restrict to the region $\Omega$ with corresponding boundary $\partial\Omega$, which is the first Gribov horizon, where the first vanishing eigenvalue of the Faddeev–Popov operator (7) appears [7]. Within the region $\Omega$ the Faddeev–Popov operator is positive definite, i.e. $\mathcal{M}^{ab} > 0$. Quite obviously,

this restriction to the first Gribov region can be motivated only if every gauge orbit passes through it. It was shown by Gribov that this is certainly the case for gauge potentials "sufficiently close" to the boundary $\partial\Omega$ [7], whereas the proof for general configurations was presented in [9]. Nevertheless, we should also mention that the Gribov region itself is also not free from gauge copies [9–12]. To avoid the presence of these additional copies, a further restriction to a smaller region $\Lambda$, known as the fundamental modular region, should be implemented. Nevertheless, the implementation of the restriction of the domain of integration to $\Lambda$ proves to be a quite difficult task, which, to our knowledge, has not yet been accomplished. Recently, it has been argued that the additional copies existing inside $\Omega$ might be irrelevant when computing expectation values, meaning that averages calculated over $\Lambda$ or $\Omega$ should give the same value [13].

Using a semiclassical argument [7], Gribov implemented the restriction to the region $\Omega$. Essentially, his argument relied on the fact that the (Fourier transform of the) inverse of the Faddeev–Popov operator, which is nothing else than the ghost propagator, encounters no poles elsewhere than at the origin $k^2 = 0$. This amounts to saying that the operator $\mathcal{M}^{ab}$ itself does not vanish, except at the horizon. By using this "no pole condition", we are assured that the considered gauge potentials remain inside the first Gribov region[1], and as such at least the set of copies obtained via (6) is already excluded from the game.

This restriction has many important consequences for the infrared behavior of the propagators. The gluon propagator turns out to be suppressed in the infrared, while the ghost propagator gets enhanced [7]. Moreover, it can also be shown that the gluon propagator exhibits a violation of positivity in its spectral density representation, a sign that the gluon cannot be a physical observable anymore; see [14–17] and references therein. It is interesting to mention that lattice simulations of the Landau gauge propagators have revealed evidence for this suppression, respectively, enhancement; see e.g. [18–26]. Another consequence of the Gribov restriction is the "infrared freezing" of the strong coupling constant, i.e. $\alpha_s(p^2)$ tends to a constant as $p^2$ goes to zero, see [14, 27] and references therein. Again, this behavior is in qualitative agreement with lattice data [22–24] as well as with the results obtained from the analysis of the Schwinger–Dyson equations [28–37].

It might be clear that the restriction to the Gribov region $\Omega$ could be of great relevance for a better understanding of the infrared region of gauge theories. This belief is further supported by the Kugo–Ojima confinement criterion [38], which, in the case of the Landau gauge, turns out to rely on a ghost propagator diverging stronger than $\frac{1}{p^2}$ [39]. This feature is also present when the restriction to the Gribov region is implemented, yielding in fact a ghost propagator developing a $\frac{1}{p^4}$ singularity.

Important progress on the restriction to the Gribov region $\Omega$ was accomplished by Zwanziger in the

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[1] Albeit that this restriction cannot be implemented exactly, but only in an order by order expansive way.

papers [40, 41]. The restriction to $\Omega$ was implemented through the introduction of a nonlocal horizon function appearing in the Boltzmann weight defining the Euclidean Yang–Mills measure. According to [40, 41], the starting Yang–Mills measure in the Landau gauge is given by

$$\mathrm{d}\mu_\gamma = DADcD\overline{c}Dbe^{-\left(S+\gamma^4 H\right)}, \qquad (8)$$

where the starting action is

$$S = S_{\mathrm{YM}} + S_{\mathrm{gf}}, \qquad (9)$$

with $S_{\mathrm{gf}}$ the gauge-fixing action given by

$$S_{\mathrm{gf}} = \int \mathrm{d}^4x \left(b^a \partial_\mu A_\mu^a + \overline{c}^a \partial_\mu D_\mu^{ab} c^b\right), \qquad (10)$$

where the auxiliary field $b^a$ is a Lagrange multiplier enforcing the Landau gauge (5), $(\overline{c}^a, c^a)$ are the Faddeev–Popov ghost fields, and

$$H = \int \mathrm{d}^4x h(x) = g^2 \int \mathrm{d}^4x f^{abc} A_\mu^b \left(\mathcal{M}^{-1}\right)^{ad} f^{dec} A_\mu^e, \qquad (11)$$

is the so-called horizon function, which implements the restriction to the Gribov region $\Omega$. We recognize that $H$ is nonlocal. The massive Gribov parameter $\gamma$ is fixed by the horizon condition

$$\langle h(x) \rangle = 4 \left(N^2 - 1\right), \qquad (12)$$

where the expectation value $\langle h(x) \rangle$ has to be evaluated with the measure (8). To the first order, the horizon condition (12) becomes, in $d$ dimensions,

$$1 = \frac{N(d-1)}{4} g^2 \int \frac{\mathrm{d}^d q}{(2\pi)^d} \frac{1}{q^4 + 2Ng^2\gamma^4}. \qquad (13)$$

This equation coincides with the original gap equation derived by Gribov for the parameter $\gamma$ [7].

We shall rely on the path integral formalism, so that we can localize the horizon function (11) by means of a pair of complex bosonic vector fields [41], $\left(\phi_\mu^{ab}, \overline{\phi}_\mu^{ab}\right)$, according to

$$\mathrm{e}^{-S_H} = \int D\overline{\phi}D\phi \, (\det \mathcal{M})^f \exp\left\{ -\int \mathrm{d}^4x \left[\overline{\phi}_\mu^{ac} \mathcal{M}^{ab} \phi_\mu^{bc} \right. \right.$$
$$\left. \left. + \gamma^2 g f^{abc} \left(\phi_\mu^{ac} - \overline{\phi}_\mu^{ac}\right) A_\mu^b \right] \right\}, \qquad (14)$$

where the determinant, $(\det \mathcal{M})^f$, takes into account the Jacobian arising from the integration over $\left(\phi_\mu^{ab}, \overline{\phi}_\mu^{ab}\right)$, and

$$f = D(N^2 - 1) = 4(N^2 - 1), \qquad (15)$$

with $D = 4$ the dimension of the Euclidean space time, and $N$ the dimension of the gauge group. This determinant can also be localized by means of suitable anticommuting complex vector fields $\left(\omega_\mu^{ab}, \overline{\omega}_\mu^{ab}\right)$, namely

$$(\det \mathcal{M})^f = \int D\overline{\omega}D\omega \exp\left[-\int \mathrm{d}^4x \left(-\overline{\omega}^{ac} \mathcal{M}^{ab} \omega_\mu^{bc}\right)\right]. \qquad (16)$$

Henceforth, the nonlocal action $S_H$ is transformed into a local one given by

$$S_H^{\mathrm{Local}} = S_{\phi\omega} + S_\gamma, \qquad (17)$$

where

$$S_{\phi\omega} = \int \mathrm{d}^4x \left(\overline{\phi}_\mu^{ac} \mathcal{M}^{ab} \phi_\mu^{bc} - \overline{\omega}_\mu^{ac} \mathcal{M}^{ab} \omega_\mu^{bc}\right), \qquad (18)$$

and

$$S_\gamma = \gamma^2 \int \mathrm{d}^4x \, g f^{abc} \left(\phi_\mu^{ac} - \overline{\phi}_\mu^{ac}\right) A_\mu^b. \qquad (19)$$

As was shown in [14, 40–42], the resulting local action turns out to be renormalizable to all orders of perturbation theory. This is a point of great importance, as it allows for a consistent and order by order improvable framework to calculate relevant quantities when the restriction to the Gribov region $\Omega$ is taken into account.

A second point that motivated this paper is the issue of the dynamical mass generation in gauge theories, and, related to it, that of the $\frac{1}{q^2}$ power corrections. A few years ago, in a series of papers, Zakharov et al. questioned the common wisdom that $\frac{1}{q^2}$ power corrections cannot enter gauge invariant observables, as local gauge invariant operators of mass dimension two do not exist. This is a reflection of the fact that one cannot add a renormalizable mass operator for the gauge fields to the Yang–Mills action, at least not when the Higgs mechanism and associated symmetry breaking are not considered. However, by using QCD sum rules, it was advocated in [43] that an effective gluon mass could account for the $\frac{1}{q^2}$ corrections, leading to an acceptable phenomenology. The underlying condensate was proposed to be the gauge invariant quantity [44, 45]

$$\langle A_{\mathrm{min}}^2 \rangle \equiv \min_{U \in \mathrm{SU}(N)} \frac{1}{VT} \int \mathrm{d}^4x \left\langle \left(A_\mu^U\right)^2 \right\rangle, \qquad (20)$$

which originates from a highly nonlocal operator, since [46]

$$A_{\mathrm{min}}^2 = \frac{1}{2} \int \mathrm{d}^4x \left[ A_\mu^a \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right) A_\nu^a \right.$$
$$\left. - g f^{abc} \left(\frac{\partial_\nu}{\partial^2} \partial A^a\right) \left(\frac{1}{\partial^2} \partial A^b\right) A_\nu^c \right] + O(A^4),$$

and therefore it falls beyond the OPE applicability. The interest was especially focused on the Landau gauge, since then the operator $A_{\mathrm{min}}^2$ reduces to the local quantity $A_\mu^2$. An effective potential for $\langle A_\mu^2 \rangle$ was calculated up to two loops in [47, 48], giving evidence for a nonvanishing condensate and consequent effective gluon mass $m^2 \propto \langle A_\mu^2 \rangle$. Determining a sensible effective potential for a local composite operator (LCO) is a nontrivial task, but nevertheless it was dealt with in [47] based on the method developed in [49]. The renormalizability of the so-called LCO method was proven in [50] to all orders of perturbation theory in the case of $A_\mu^2$ in the Landau gauge.

Unfortunately, the gauge invariance of $A_{\mathrm{min}}^2$ is, strictly speaking, only ensured when the *absolute* minimum of

$A^2_{\min}$ along the gauge orbit has been reached, a highly difficult task due to the presence of Gribov copies. Moreover, it is unclear what can be done with this operator beyond the Landau gauge. In other gauges, other renormalizable and condensing dimension two operators exist, but these are explicitly gauge parameter or even ghost dependent; see [51] for an overview. In e.g. the maximal Abelian gauge, an effective mass was found for the off-diagonal gluons only [52], which is qualitatively consistent with the available lattice data [53, 54].

Let us also mention that effective gluon masses have been studied in the past from a theoretical, phenomenological and numerical viewpoint; see [20, 47, 55–62] for a far from exhaustive list.

Taking all this into account, it seems to be a worthy task to look for other potential candidates, which could be at the origin of the dynamical mass generation and related $\frac{1}{q^2}$ power corrections. It would also be favorable to start from a gauge invariant operator. The candidate we already investigated in [46, 63] is the nonlocal operator[2]

$$\mathcal{O} = (VT)^{-1} \int \mathrm{d}^4x\, F^a_{\mu\nu} \left[(D^2)^{-1}\right]^{ab} F^b_{\mu\nu}, \qquad (21)$$

which can be coupled to the Yang–Mills action via a nonlocal mass term

$$S_{\mathcal{O}} = -\frac{m^2}{4} \int \mathrm{d}^4x\, F^a_{\mu\nu} \left[(D^2)^{-1}\right]^{ab} F^b_{\mu\nu}, \qquad (22)$$

where $m$ is a mass parameter, and $[(D^2)^{-1}]^{ab}$ is the inverse of the covariant Laplacian

$$\begin{aligned} D^2 \equiv D^{ac}_\mu D^{cb}_\mu = {}& \partial^2 \delta^{ab} - 2gf^{abc}A^c_\mu \partial_\mu \\ & - gf^{abc}\partial_\mu A^c_\mu + g^2 f^{acd}f^{cbe}A^d_\mu A^e_\mu. \end{aligned} \qquad (23)$$

Analogously to what has been done in the case of the nonlocal horizon function (11), the gauge invariant mass operator (22) can be localized with the help of a pair of complex bosonic antisymmetric tensor fields in the adjoint representation [46], $\left(B^a_{\mu\nu}, \overline{B}^a_{\mu\nu}\right)$,

$$\begin{aligned} \mathrm{e}^{-S_{\mathcal{O}}} = {}& \int D\overline{B}DB\, (\det D^2)^{f'} \\ & \mathrm{e}^{\left\{-\frac{1}{4}\int \mathrm{d}^4x\, \left[\overline{B}^a_{\mu\nu}D^{ac}_\sigma D^{cb}_\sigma B^b_{\mu\nu} + im(B^a_{\mu\nu}-\overline{B}^a_{\mu\nu})F^a_{\mu\nu}\right]\right\}}. \end{aligned} \qquad (24)$$

Here we have

$$f' = \frac{D(D-1)}{2} = 6, \qquad (25)$$

and, like in the case of the horizon function, the determinant, $(\det D^2)^{f'}$, can be localized using a pair of anticommuting antisymmetric complex tensor fields $\left(G^a_{\mu\nu}, \overline{G}^a_{\mu\nu}\right)$,

according to

$$(\det D^2)^{f'} = \int D\overline{G}DG\, \mathrm{e}^{\left[-\frac{1}{4}\int \mathrm{d}^4x\, \left(\overline{G}^a_{\mu\nu}D^{ac}_\sigma D^{cb}_\sigma G^b_{\mu\nu}\right)\right]}. \quad (26)$$

Then, the action $S_{\mathcal{O}}$ gets replaced by its local version given by

$$S_{\mathcal{O}}^{\mathrm{Local}} = S_{\mathrm{BG}} + S_m, \qquad (27)$$

where

$$S_{\mathrm{BG}} = \frac{1}{4} \int \mathrm{d}^4x\, \left(\overline{B}^a_{\mu\nu}D^{ac}_\sigma D^{cb}_\sigma B^b_{\mu\nu} - \overline{G}^a_{\mu\nu}D^{ac}_\sigma D^{cb}_\sigma G^b_{\mu\nu}\right), \qquad (28)$$

and

$$S_m = \frac{im}{4} \int \mathrm{d}^4x\, (B^a_{\mu\nu} - \overline{B}^a_{\mu\nu})F^a_{\mu\nu}. \qquad (29)$$

We underline the fact that an initially nonlocal operator can be cast into a local form [65]. In the case of $A^2_{\min}$, this would not be possible, as it is a infinite series of different nonlocal operators. Once we arrive at a local action, we can investigate e.g. the renormalizability to all orders by means of algebraic methods, the canonical quantization, the explicit calculation of the renormalization factors, etc.

The goal of this paper is to study the massive action (22) when the restriction to the Gribov region $\Omega$ is implemented à la Zwanziger. Since the extended action $S_{\mathrm{YM}} + S_{\mathcal{O}}$ is gauge invariant, we might expect that the procedure of further restricting the domain of integration will have no influence on the renormalizability. This will be explicitly confirmed. In a future stage of research, one can start searching for the value of the Gribov parameter $\gamma$ as well as the dynamically generated mass $m$. We recall here that the Gribov–Zwanziger action itself can also be used to mimic $\frac{1}{q^2}$ corrections, as explicitly discussed in [27]. As a future endeavor, it would be worthwhile to study physical correlators with our action, and find whether the Gribov and/or mass parameter $m$ are a potential source of such power corrections.

Let us return to the content of this paper, which is organized as follows. In Sect. 2, we introduce all the necessary sources in order to find a suitable starting action. The set of Ward identities defining this action is presented in Sect. 3, while in Sect. 4 we compute several useful (anti-) commutation relations between the linearized symmetry operators. These are used in Sect. 5 in order to construct the most general allowed invariant counterterm. In Sect. 6, we confirm the renormalizability, since we shall be able to reabsorb all the allowed counterterms in the starting action by introducing suitable bare quantities. In Sect. 7 we discuss a few properties of the physical action, which is obtained from the starting action by setting the sources equal to their physically relevant values. The process of giving specific values to the sources breaks the BRST invariance. In Sect. 8, we discuss the associated breaking of the Slavnov–Taylor identity, and we comment on the fact that in most cases this breaking becomes harmless for the identities derivable between a large

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[2] This particular operator was already proposed in $3D$ gauge theories [64].

class of Green functions. Finally, Sect. 9 is devoted to the conclusions.

# 2 Identification of the complete classical action $\Sigma$

We shall start with the following local action, as it was obtained in the introduction:

$$S_{\text{Local}} = S_{\text{YM}} + S_{\text{gf}} + S_{\text{H}}^{\text{Local}} + S_{\mathcal{O}}^{\text{Local}}. \qquad (30)$$

As we wish to discuss the renormalizability, we should try to establish as many symmetries as possible. These symmetries can then be translated into Ward identities. As we are dealing with a gauge theory that is to be gauge fixed, we expect to find a BRST invariance and consequent Slavnov–Taylor identity. All these identities are a powerful tool in constructing the most general allowed counterterm [66]. If this counterterm can be reabsorbed in the original action through the introduction of bare quantities, we are able to conclude that the starting action is renormalizable. If not, we could still try to identify a more general starting action that is renormalizable. This has been discussed in extenso already in [46, 63] when analyzing the nonlocal mass term (22).

## 2.1 BRST invariance

In order to find the BRST invariance of the resulting local theory, given by (30), we proceed as in [41, 46] and consider at first the particular case when $\gamma = m = 0$, i.e.,

$$
\begin{aligned}
S_{\text{Local}}^{\gamma=m=0} &= S_{\text{YM}} + S_{\text{gf}} + S_{\text{H}}^{\text{Local},\gamma=0} + S_{\mathcal{O}}^{\text{Local},m=0} \\
&= S_{\text{YM}} + S_{\text{gf}} + S_{\phi\omega} + S_{\text{BG}}. \qquad (31)
\end{aligned}
$$

In this case, we have actually introduced nothing more than two unity factors, written as

$$
\begin{aligned}
1 &= \int D\overline{\phi} D\phi D\overline{\omega} D\omega \\
&\exp\left[ -\int \mathrm{d}^4x \left( \overline{\phi}_\mu^{ac} \mathcal{M}^{ab} \phi_\mu^{bc} - \overline{\omega}_\mu^{ac} \mathcal{M}^{ab} \omega_\mu^{bc} \right) \right], \\
1 &= \int D\overline{B} DB D\overline{G} DG \\
&\exp\left[ -\frac{1}{4} \int \mathrm{d}^4x \left( \overline{B}_{\mu\nu}^a D_\sigma^{ac} D_\sigma^{cb} B_{\mu\nu}^b - \overline{G}_{\mu\nu}^a D_\sigma^{ac} D_\sigma^{cb} G_{\mu\nu}^b \right) \right]. \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (32)
\end{aligned}
$$

Nevertheless, the action (31) may be written in a BRST invariant fashion. To see this, let us first introduce the following nilpotent BRST transformation

$$
\begin{aligned}
sA_\mu^a &= -D_\mu^{ab} c^b, \\
sc^a &= \frac{g}{2} f^{abc} c^b c^c,
\end{aligned}
$$

$$
\begin{aligned}
sB_{\mu\nu}^a &= g f^{abc} c^b B_{\mu\nu}^c + G_{\mu\nu}^a, \\
sG_{\mu\nu}^a &= g f^{abc} c^b G_{\mu\nu}^c, \\
s\overline{G}_{\mu\nu}^a &= g f^{abc} c^b \overline{G}_{\mu\nu}^c + \overline{B}_{\mu\nu}^a, \\
s\overline{B}_{\mu\nu}^a &= g f^{abc} c^b \overline{B}_{\mu\nu}^c, \\
s\overline{c}^a &= b^a, \\
sb^a &= 0, \\
s\phi_\mu^{ab} &= \omega_\mu^{ab}, \\
s\omega_\mu^{ab} &= 0, \\
s\overline{\omega}_\mu^{ab} &= \overline{\phi}_\mu^{ab}, \\
s\overline{\phi}_\mu^{ab} &= 0, \\
s^2 &= 0. \qquad (33)
\end{aligned}
$$

Now, let $S_0$ be the action defined by

$$
\begin{aligned}
S_0 = S_{\text{YM}} + s \int \mathrm{d}^4x \ \big( &\overline{c}^a \partial_\mu A_\mu^a + \overline{\omega}_\mu^{ac} \mathcal{M}^{ab} \phi_\mu^{bc} \\
&+ \overline{G}_{\mu\nu}^a D_\sigma^{ac} D_\sigma^{cb} B_{\mu\nu}^b \big), \qquad (34)
\end{aligned}
$$

which satifies

$$sS_0 = 0. \qquad (35)$$

Applying the BRST transformations (33) and recalling that the Faddeev–Popov operator, $\mathcal{M}^{ab}$, is given by (7), we obtain

$$S_0 = S_{\text{Local}}^{\gamma=m=0} + \int \mathrm{d}^4x \ \overline{\omega}_\mu^{ac} \partial_\nu \big( g f^{abd} \phi_\mu^{bc} D_\nu^{de} c^e \big). \qquad (36)$$

Following [41] one can show that $S_0$ and $S_{\text{Local}}^{\gamma=m=0}$ are equivalent. More precisely, one may transform $S_{\text{Local}}^{\gamma=m=0}$ into $S_0$ by performing the following shift in the variable $\omega_\mu^{ac}$,

$$\omega_\mu^{ac} \rightarrow \omega_\mu^{ac} - \left(\mathcal{M}^{-1}\right)^{ab} \partial_\nu \big( g f^{bed} \phi_\mu^{ec} D_\nu^{dn} c^n \big), \qquad (37)$$

and keeping in mind that the corresponding Jacobian turns out to be field independent. Thus, the following equivalence holds:

$$\int D\Phi \, \mathrm{e}^{-S_0} = \int D\Phi \, \mathrm{e}^{-S_{\text{Local}}^{\gamma=m=0}}, \qquad (38)$$

where $\Phi$ is a shorthand for all the fields. Now, let us reintroduce the term $S_\gamma$, given by (19), while $S_m$ remains absent. It is easy to show, using the BRST transformations (33), that $S_\gamma$ may be rewritten as

$$
\begin{aligned}
S_\gamma = \gamma^2 \int \mathrm{d}^4x \ \Big[ &g f^{abc} \phi_\mu^{ac} A_\mu^b - s \big( g f^{abc} \overline{\omega}_\mu^{ac} A_\mu^b \big) \\
&+ g f^{abc} \overline{\omega}_\mu^{ac} D_\mu^{bd} c^d \Big]. \qquad (39)
\end{aligned}
$$

The last term can be eliminated by means of the change of variables

$$\omega_\mu^{bc} \rightarrow \omega_\mu^{bc} + \gamma^2 \left(\mathcal{M}^{-1}\right)^{bd} g f^{dec} D_\mu^{en} c^n. \qquad (40)$$

Furthermore, we notice that, thanks to the fact that the integral of a total derivative vanishes, the following expression for $S_\gamma$ holds:

$$S_\gamma = -\gamma^2 \int \mathrm{d}^4x \left[ D_\mu^{ab} \phi_\mu^{ba} - s\left( D_\mu^{ab} \overline{\omega}_\mu^{ba} \right) \right]. \quad (41)$$

Nevertheless, the action,

$$S_0 + S_\gamma, \quad (42)$$

is not yet BRST invariant. This point can be dealt with by means of the introduction of a pair of BRST doublets of local external sources [41], $\left( M_{\mu\nu}^{ab}, N_{\mu\nu}^{ab} \right)$ and $\left( \overline{M}_{\mu\nu}^{ab}, \overline{N}_{\mu\nu}^{ab} \right)$, which transform as

$$sM_{\mu\nu}^{ab} = -N_{\mu\nu}^{ab}, \quad sN_{\mu\nu}^{ab} = 0,$$
$$s\overline{N}_{\mu\nu}^{ab} = -\overline{M}_{\mu\nu}^{ab}, \quad s\overline{M}_{\mu\nu}^{ab} = 0. \quad (43)$$

As pointed out in [41], the introduction of these external sources allows us to promote expression (42) to a BRST invariant action. In fact, let $S_{\text{sources}}$ be the action

$$S_{\text{sources}} = s \int \mathrm{d}^4x \left( \overline{N}_{\mu\nu}^{ac} D_\mu^{ab} \phi_\nu^{bc} - M_{\mu\nu}^{ac} D_\mu^{ab} \overline{\omega}_\nu^{bc} \right)$$
$$= \int \mathrm{d}^4x \left[ -\overline{M}_{\mu\nu}^{ac} D_\mu^{ab} \phi_\nu^{bc} - \overline{N}_{\mu\nu}^{ac} s\left( D_\mu^{ab} \phi_\nu^{bc} \right) \right.$$
$$\left. + N_{\mu\nu}^{ac} D_\mu^{ab} \overline{\omega}_\nu^{bc} - M_{\mu\nu}^{ac} s\left( D_\mu^{ab} \overline{\omega}_\nu^{bc} \right) \right], \quad (44)$$

which obviously satisfies

$$sS_{\text{sources}} = 0. \quad (45)$$

When the sources $\left( M_{\mu\nu}^{ab}, \overline{M}_{\mu\nu}^{ab}, N_{\mu\nu}^{ab}, \overline{N}_{\mu\nu}^{ab} \right)$ attain their physical values [41], defined by

$$\overline{M}_{\mu\nu}^{ab} \Big|_{\text{phys}} = -M_{\mu\nu}^{ab} \Big|_{\text{phys}} = -\gamma^2 \delta^{ab} \delta_{\mu\nu},$$
$$N_{\mu\nu}^{ab} \Big|_{\text{phys}} = \overline{N}_{\mu\nu}^{ab} \Big|_{\text{phys}} = 0, \quad (46)$$

it immediately follows that

$$S_{\text{sources}}|_{\text{phys}} = S_\gamma = -\gamma^2 \int \mathrm{d}^4x \left[ D_\mu^{ab} \phi_\mu^{ba} - s\left( D_\mu^{ab} \overline{\omega}_\mu^{ba} \right) \right]. \quad (47)$$

One sees thus that the use of the external sources $\left( M_{\mu\nu}^{ab}, \overline{M}_{\mu\nu}^{ab}, N_{\mu\nu}^{ab} \text{ and } \overline{N}_{\mu\nu}^{ab} \right)$ enables us to introduce an extended action $\Sigma_0$, given by

$$\Sigma_0 = S_0 + S_{\text{sources}}, \quad (48)$$

which enjoys the important property of being BRST invariant,

$$s\Sigma_0 = 0, \quad (49)$$

while reducing to expression (42) when the sources attain their physical values, given by (46). Recapitulating, we have rewritten

$$\int DADcD\overline{c}Db\, \mathrm{e}^{-S+\gamma^4 H}$$
$$= \int DADcD\overline{c}DbD\overline{\phi}D\phi D\overline{\omega}D\omega D\mathrm{e}^{-S_0-S_\gamma}. \quad (50)$$

It is then easily shown, upon combination of (11), (12), (42) and (50), that the horizon condition is implemented by requiring that

$$\frac{\partial \Gamma_{\text{GZ}}}{\partial \gamma^2} = 0, \quad \text{with } \gamma^2 \neq 0, \quad (51)$$

whereby $\Gamma_{\text{GZ}}$ is the Gribov–Zwanziger effective action defined by

$$\mathrm{e}^{-\Gamma_{\text{GZ}}} = \int DADcD\overline{c}DbD\overline{\phi}D\phi D\overline{\omega}D\omega\, \mathrm{e}^{-S_0-S_\gamma}. \quad (52)$$

To continue, let us analyze the term $S_m$, given by (29). This term is, just as $S_{\mathcal{O}}^{\text{Local}}$ in (27), left invariant by the gauge transformations [46]

$$\delta A_\mu^a = -D_\mu^{ab} \theta^b,$$
$$\delta B_{\mu\nu}^a = g f^{abc} \theta^b B_{\mu\nu}^c,$$
$$\delta G_{\mu\nu}^a = g f^{abc} \theta^b G_{\mu\nu}^c,$$
$$\delta \overline{G}_{\mu\nu}^a = g f^{abc} \theta^b \overline{G}_{\mu\nu}^c,$$
$$\delta \overline{B}_{\mu\nu}^a = g f^{abc} \theta^b \overline{B}_{\mu\nu}^c, \quad (53)$$

where $\theta^a$ is the parameter of the gauge transformation, but it is not invariant by the BRST transformations (33). This problem can be solved in a way equivalent as done in the case of $S_\gamma$. This time we will introduce a pair of BRST doublets of external sources, $(U_{\alpha\beta\mu\nu}, V_{\alpha\beta\mu\nu})$ and $(\overline{U}_{\alpha\beta\mu\nu}, \overline{V}_{\alpha\beta\mu\nu})$, transforming as

$$sV_{\alpha\beta\mu\nu} = U_{\alpha\beta\mu\nu}, \quad sU_{\alpha\beta\mu\nu} = 0,$$
$$s\overline{U}_{\alpha\beta\mu\nu} = \overline{V}_{\alpha\beta\mu\nu}, \quad s\overline{V}_{\alpha\beta\mu\nu} = 0. \quad (54)$$

Hence, by considering the following term

$$S'_{\text{sources}} = s \int \mathrm{d}^4x \left( V_{\alpha\beta\mu\nu} \overline{G}_{\alpha\beta}^a - \overline{U}_{\alpha\beta\mu\nu} B_{\alpha\beta}^a \right) F_{\mu\nu}^a,$$

the term $S'_{\text{sources}}$ reduces to $S_m$ of (29) when the sources $\left( U_{\alpha\beta\mu\nu}, \overline{U}_{\alpha\beta\mu\nu}, V_{\alpha\beta\mu\nu}, \overline{V}_{\alpha\beta\mu\nu} \right)$ attain the subsequent physical values

$$\overline{V}_{\alpha\beta\mu\nu}|_{\text{phys}} = V_{\alpha\beta\mu\nu}|_{\text{phys}} = -\frac{im}{2}(\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu}),$$
$$U_{\alpha\beta\mu\nu}|_{\text{phys}} = \overline{U}_{\alpha\beta\mu\nu}|_{\text{phys}} = 0. \quad (55)$$

These sources enable us to define an action $\Sigma_1$ by

$$\Sigma_1 = \Sigma_0 + S'_{\text{sources}}, \quad (56)$$

in such way that

$$s\Sigma_1 = 0. \quad (57)$$

## 2.2 The global U($f$) and U($f'$) symmetries

In addition to the BRST invariance the action $\Sigma_1$ displays the global symmetries U($f$), $f = 4(N^2 - 1)$ and U($f'$), $f' = 6$, respectively expressed by

$$\mathcal{Q}_{\mu\nu}^{ab}(\Sigma_1) \equiv \int \mathrm{d}^4x \left( \phi_\mu^{ca} \frac{\delta\Sigma_1}{\delta\phi_\nu^{cb}} - \overline{\phi}_\nu^{cb} \frac{\delta\Sigma_1}{\delta\overline{\phi}_\mu^{ca}} + \omega_\mu^{ca} \frac{\delta\Sigma_1}{\delta\omega_\nu^{cb}} \right.$$
$$- \overline{\omega}_\nu^{cb} \frac{\delta\Sigma_1}{\delta\overline{\omega}_\nu^{ca}} + M_{\sigma\mu}^{ca} \frac{\delta\Sigma_1}{\delta M_{\sigma\nu}^{cb}} - \overline{M}_{\sigma\nu}^{cb} \frac{\delta\Sigma_1}{\delta\overline{M}_{\sigma\mu}^{ca}}$$
$$\left. + N_{\sigma\mu}^{ca} \frac{\delta\Sigma_1}{\delta N_{\sigma\nu}^{cb}} - \overline{N}_{\sigma\nu}^{cb} \frac{\delta\Sigma_1}{\delta\overline{N}_{\sigma\mu}^{ca}} \right) = 0 , \tag{58}$$

and

$$\mathcal{Q}_{\alpha\beta\mu\nu}(\Sigma_1) \equiv \int \mathrm{d}^4x \left( B_{\alpha\beta}^a \frac{\delta\Sigma_1}{\delta B_{\mu\nu}^a} - \overline{B}_{\mu\nu}^a \frac{\delta\Sigma_1}{\delta\overline{B}_{\alpha\beta}^a} + G_{\alpha\beta}^a \frac{\delta\Sigma_1}{\delta G_{\mu\nu}^a} \right.$$
$$- \overline{G}_{\mu\nu}^a \frac{\delta\Sigma_1}{\delta\overline{G}_{\alpha\beta}^a} + U_{\alpha\beta\sigma\rho} \frac{\delta\Sigma_1}{\delta U_{\mu\nu\sigma\rho}}$$
$$- \overline{U}_{\mu\nu\sigma\rho} \frac{\delta\Sigma_1}{\delta\overline{U}_{\alpha\beta\sigma\rho}} + V_{\alpha\beta\sigma\rho} \frac{\delta\Sigma_1}{\delta V_{\mu\nu\sigma\rho}}$$
$$\left. - \overline{V}_{\mu\nu\sigma\rho} \frac{\delta\Sigma_1}{\delta\overline{V}_{\alpha\beta\sigma\rho}} \right) = 0 . \tag{59}$$

The presence of the global invariances U($f$) and U($f'$) means that one can make use [41, 46] of the composite indices $I \equiv (a, \mu)$, $I = 1, \ldots, f$, and $i \equiv (\mu, \nu)$, $i = 1, \ldots, f'$. Specifically, setting

$$\left( \phi_I^a, \overline{\phi}_I^a, \omega_I^a, \overline{\omega}_I^a \right) \equiv \left( \phi_\mu^{ab}, \overline{\phi}_\mu^{ab}, \omega_\mu^{ab}, \overline{\omega}_\mu^{ab} \right) ,$$
$$\left( M_{\mu I}^a, \overline{M}_{\mu I}^a, N_{\mu I}^a, \overline{N}_{\mu I}^a \right) \equiv \left( M_{\mu\nu}^{ab}, \overline{M}_{\mu\nu}^{ab}, N_{\mu\nu}^{ab}, \overline{N}_{\mu\nu}^{ab} \right) , \tag{60}$$

and

$$\left( B_i^a, \overline{B}_i^a, G_i^a, \overline{G}_i^a \right) \equiv \frac{1}{2} \left( B_{\mu\nu}^a, \overline{B}_{\mu\nu}^a, G_{\mu\nu}^a, \overline{G}_{\mu\nu}^a \right) ,$$
$$(U, \overline{U}, V, \overline{V})_{i\mu\nu} \equiv \frac{1}{2} (U, \overline{U}, V, \overline{V})_{\alpha\beta\mu\nu} , \tag{61}$$

we rewrite $\Sigma_1$ as

$$\Sigma_1 = S_{\mathrm{YM}} + \int \mathrm{d}^4x \left\{ b^a \, \partial_\mu A_\mu^a + \overline{c}^a \, \partial_\mu D_\mu^{ab} c^b + \overline{\phi}_I^a \mathcal{M}^{ab} \phi_I^b \right.$$
$$- \overline{\omega}_I^a \mathcal{M}^{ab} \omega_I^b + g f^{abc} \overline{\omega}_I^a \partial_\mu \left( \phi_I^b D_\mu^{cd} c^d \right) - \overline{M}_{\mu I}^a \, D_\mu^{ab} \phi_I^b$$
$$- \overline{N}_{\mu I}^a \left[ D_\mu^{ab} \omega_I^b + g f^{abc} \phi_I^b D_\mu^{cd} c^d \right] + N_{\mu I}^a \, D_\mu^{ab} \overline{\omega}_I^b$$
$$- M_{\mu I}^a \left[ D_\mu^{ab} \overline{\phi}_I^b - g f^{abc} \overline{\omega}_I^a D_\mu^{cd} c^d \right] + \overline{B}_i^a D_\mu^{ab} D_\mu^{bc} B_i^c$$
$$- \overline{G}_i^a D_\mu^{ab} D_\mu^{bc} G_i^c + F_{\mu\nu}^a \left( \overline{U}_{i\mu\nu} G_i^a + V_{i\mu\nu} \overline{B}_i^a \right.$$
$$\left. \left. - \overline{V}_{i\mu\nu} B_i^a + U_{i\mu\nu} \overline{G}_i^a \right) \right\} . \tag{62}$$

For the symmetry generators, we have

$$\mathcal{Q}_{IJ} \equiv \int \mathrm{d}^4x \left( \phi_I^a \frac{\delta}{\delta\phi_J^a} - \overline{\phi}_J^a \frac{\delta}{\delta\overline{\phi}_I^a} + \omega_I^a \frac{\delta}{\delta\omega_J^a} - \overline{\omega}_J^a \frac{\delta}{\delta\overline{\omega}_I^a} \right.$$
$$+ M_{\mu I}^a \frac{\delta}{\delta M_{\mu J}^a} - \overline{M}_J^a \frac{\delta}{\delta\overline{M}_I^a}$$
$$\left. + N_{\mu I}^a \frac{\delta}{\delta N_{\mu J}^a} - \overline{N}_{\mu J}^a \frac{\delta}{\delta\overline{N}_{\mu I}^a} \right) , \tag{63}$$

and

$$\mathcal{Q}_{ij} \equiv \int \mathrm{d}^4x \left( B_i^a \frac{\delta}{\delta B_j^a} - \overline{B}_j^a \frac{\delta}{\delta\overline{B}_i^a} + G_i^a \frac{\delta}{\delta G_j^a} - \overline{G}_j^a \frac{\delta}{\delta\overline{G}_i^a} \right.$$
$$+ U_{i\mu\nu} \frac{\delta}{\delta U_{j\mu\nu}} - \overline{U}_{j\mu\nu} \frac{\delta}{\delta\overline{U}_{i\mu\nu}}$$
$$\left. + V_{i\mu\nu} \frac{\delta}{\delta V_{j\mu\nu}} - \overline{V}_{j\mu\nu} \frac{\delta}{\delta\overline{V}_{i\mu\nu}} \right) . \tag{64}$$

By means of the trace of these operators the $I(i)$-valued fields turn out to possess an additional quantum number, displayed in Tables 1 and 2, together with the dimension and the ghost number.

## 2.3 The complete classical action $\Sigma$

We proceed by establishing the complete set of Ward identities, which will enable us to analyze the renormalizability of the theory to all orders. Let us first identify the final complete action to start with. For this purpose, we need to introduce additional external sources ($\Omega_\mu^a$, $L^a$, $\overline{Y}_i^a$, $Y_i^a$, $\overline{X}_i^a$, $X_i^a$) in order to define at quantum level the composite operators entering the nonlinear BRST transformations

**Table 1.** Quantum numbers of the fields

|  | $A$ | $b$ | $\overline{c}$ | $c$ | $\phi$ | $\overline{\phi}$ |
|---|---|---|---|---|---|---|
| dimension | 1 | 2 | 2 | 0 | 1 | 1 |
| ghost number | 0 | 0 | −1 | 1 | 0 | 0 |
| $Q_f$-charge | 0 | 0 | 0 | 0 | 1 | −1 |
| $Q_{f'}$-charge | 0 | 0 | 0 | 0 | 0 | 0 |
|  | $\omega$ | $\overline{\omega}$ | $B$ | $\overline{B}$ | $G$ | $\overline{G}$ |
| dimension | 1 | 1 | 1 | 1 | 1 | 1 |
| ghost number | 1 | −1 | 0 | 0 | 1 | −1 |
| $Q_f$-charge | 1 | −1 | 0 | 0 | 0 | 0 |
| $Q_{f'}$-charge | 0 | 0 | 1 | −1 | 1 | −1 |

**Table 2.** Quantum numbers of the sources

|  | $M$ | $\overline{M}$ | $N$ | $\overline{N}$ | $V$ | $\overline{V}$ | $U$ | $\overline{U}$ |
|---|---|---|---|---|---|---|---|---|
| dimension | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| ghost number | 0 | 0 | 1 | −1 | 0 | 0 | 1 | −1 |
| $Q_f$-charge | 1 | −1 | 1 | −1 | 0 | 0 | 0 | 0 |
| $Q_{f'}$-charge | 0 | 0 | 0 | 0 | 1 | −1 | 1 | −1 |

of the fields $\left(A_\mu^a, L^a, B_i^a, \overline{B}_i^a, G_i^a, \overline{G}_i^a\right)$, (33). In the present case this term reads

$$S_{\text{ext}} = s \int \mathrm{d}^4x \left(-\Omega_\mu^a A_\mu^a + L^a c^a - \overline{Y}_i^a B_i^a \right.$$
$$\left. - Y_i^a \overline{B}_i^a + \overline{X}_i^a G_i^a + X_i^a \overline{G}_i^a\right), \tag{65}$$

with

$$s\Omega_\mu^a = sL^a = 0, \tag{66}$$

and

$$sY_i^a = X_i^a,$$
$$sX_i^a = 0,$$
$$s\overline{X}_i^a = -\overline{Y}_i^a,$$
$$s\overline{Y}_i^a = 0. \tag{67}$$

The quantum numbers of the external sources $\left(\Omega_\mu^a, L^a, \overline{Y}_i^a, Y_i^a, \overline{X}_i^a, X_i^a\right)$ are displayed in Table 3. Furthermore, we have to add the extra source term $S_{\text{extra}}$ for renormalization purposes, as was explained in [41, 46]

$$S_{\text{extra}} = \int \mathrm{d}^4x \left\{ \overline{M}_{\mu I}^a M_{\mu I}^a - \overline{N}_{\mu I}^a N_{\mu I}^a \right.$$
$$+ \lambda_1 \left(\overline{B}_i^a B_i^a - \overline{G}_i^a G_i^a\right)\left(\overline{V}_{j\mu\nu} V_{j\mu\nu} - \overline{U}_{j\mu\nu} U_{j\mu\nu}\right)$$
$$+ \frac{\lambda^{abcd}}{16}\left(\overline{B}_i^a B_i^b - \overline{G}_i^a G_i^b\right)\left(\overline{B}_j^c B_j^d - \overline{G}_j^c G_j^d\right)$$
$$+ \lambda_3\left(\overline{B}_i^a G_j^a V_{i\mu\nu}\overline{U}_{j\mu\nu} + \overline{G}_i^a G_j^a U_{i\mu\nu}\overline{U}_{j\mu\nu}\right.$$
$$+ \overline{B}_i^a B_j^a V_{i\mu\nu}\overline{V}_{j\mu\nu} - \overline{G}_i^a B_j^a V_{j\mu\nu} U_{i\mu\nu}$$
$$- G_i^a B_j^a \overline{U}_{i\mu\nu}\overline{V}_{j\mu\nu} + \overline{G}_i^a \overline{B}_j^a U_{i\mu\nu} V_{j\mu\nu}$$
$$- \frac{1}{2}B_i^a B_j^a \overline{V}_{i\mu\nu}\overline{V}_{j\mu\nu} + \frac{1}{2}G_i^a G_j^a \overline{U}_{i\mu\nu}\overline{U}_{j\mu\nu}$$
$$\left. - \frac{1}{2}\overline{B}_i^a \overline{B}_j^a V_{i\mu\nu} V_{j\mu\nu} + \frac{1}{2}\overline{G}_i^a \overline{G}_j^a U_{i\mu\nu} U_{j\mu\nu}\right)$$
$$+ \chi_1\left(\overline{V}_{i\mu\nu}\partial^2 V_{i\mu\nu} - \overline{U}_{i\mu\nu}\partial^2 U_{i\mu\nu}\right)$$
$$+ \chi_2\left(\overline{V}_{i\mu\nu}\partial_\mu\partial_\alpha V_{i\nu\alpha} - \overline{U}_{i\mu\nu}\partial_\mu\partial_\alpha U_{i\nu\alpha}\right)$$
$$- \zeta\left(\overline{U}_{i\mu\nu} U_{i\mu\nu}\overline{U}_{j\alpha\beta} U_{j\alpha\beta} + \overline{V}_{i\mu\nu} V_{i\mu\nu}\overline{V}_{j\alpha\beta} V_{j\alpha\beta}\right.$$
$$\left.\left. - 2\overline{U}_{i\mu\nu} U_{i\mu\nu}\overline{V}_{j\alpha\beta} V_{j\alpha\beta}\right)\right\}, \tag{68}$$

where $\lambda_1, \lambda_3, \chi_1, \chi_2, \zeta$ are free parameters, and the gauge invariant rank 4 tensor $\lambda^{abcd}$ has the following symmetry

**Table 3.** Quantum numbers of the external sources

|  | $X$ | $\overline{X}$ | $Y$ | $\overline{Y}$ | $\Omega$ | $L$ |
|---|---|---|---|---|---|---|
| dimension | 3 | 3 | 3 | 3 | 3 | 4 |
| ghost number | 0 | −2 | −1 | −1 | −1 | −2 |
| $Q_f$-charge | 0 | 0 | 0 | 0 | 0 | 0 |
| $Q_{f'}$-charge | 1 | −1 | 1 | −1 | 0 | 0 |

properties:

$$\lambda^{abcd} = \lambda^{cdab} = \lambda^{bacd}, \tag{69}$$

and it obeys a generalized Jacobi identity

$$f^{man}\lambda^{mbcd} + f^{mbn}\lambda^{amcd} + f^{mcn}\lambda^{abmd} + f^{mdn}\lambda^{abcn} = 0. \tag{70}$$

Thus, the complete action we are looking for is

$$\Sigma = \Sigma_1 + S_{\text{ext}} + S_{\text{extra}}$$
$$= S_{\text{YM}} + \int \mathrm{d}^4x \left\{ b^a\,\partial_\mu A_\mu^a + \overline{c}^a\partial_\mu D_\mu^{ab}c^b + \overline{\phi}_I^a \mathcal{M}^{ab}\phi_I^b \right.$$
$$- \overline{\omega}_I^a \mathcal{M}^{ab}\omega_I^b + gf^{abc}\overline{\omega}_I^a \partial_\mu\left(\phi_I^b D_\mu^{cd}c^d\right) - \overline{M}_{\mu I}^a D_\mu^{ab}\phi_I^b$$
$$- \overline{N}_{\mu I}^a\left[D_\mu^{ab}\omega_I^b + gf^{abc}\phi_I^b D_\mu^{cd}c^d\right] + N_{\mu I}^a D_\mu^{ab}\overline{\omega}_I^b$$
$$- M_{\mu I}^a\left[D_\mu^{ab}\overline{\phi}_I^b - gf^{abc}\overline{\omega}_I^b D_\mu^{cd}c^d\right] + \overline{M}_{\mu I}^a M_{\mu I}^a$$
$$- \overline{N}_{\mu I}^a N_{\mu I}^a + \overline{B}_i^a D_\mu^{ab} D_\mu^{bc} B_i^c - \overline{G}_i^a D_\mu^{ab} D_\mu^{bc} G_i^c$$
$$+ F_{\mu\nu}^a\left(\overline{U}_{i\mu\nu} G_i^a + V_{i\mu\nu}\overline{B}_i^a - \overline{V}_{i\mu\nu} B_i^a + U_{i\mu\nu}\overline{G}_i^a\right)$$
$$+ \lambda_1\left(\overline{B}_i^a B_i^a - \overline{G}_i^a G_i^a\right)\left(\overline{V}_{j\mu\nu} V_{j\mu\nu} - \overline{U}_{j\mu\nu} U_{j\mu\nu}\right)$$
$$+ \frac{\lambda^{abcd}}{16}\left(\overline{B}_i^a B_i^b - \overline{G}_i^a G_i^b\right)\left(\overline{B}_j^c B_j^d - \overline{G}_j^c G_j^d\right)$$
$$+ \lambda_3\left(\overline{B}_i^a G_j^a V_{i\mu\nu}\overline{U}_{j\mu\nu} + \overline{G}_i^a G_j^a U_{i\mu\nu}\overline{U}_{j\mu\nu}\right.$$
$$+ \overline{B}_i^a B_j^a V_{i\mu\nu}\overline{V}_{j\mu\nu} - \overline{G}_i^a B_j^a V_{j\mu\nu} U_{i\mu\nu}$$
$$- G_i^a B_j^a \overline{U}_{i\mu\nu}\overline{V}_{j\mu\nu} + \overline{G}_i^a \overline{B}_j^a U_{i\mu\nu} V_{j\mu\nu}$$
$$- \frac{1}{2}B_i^a B_j^a \overline{V}_{i\mu\nu}\overline{V}_{j\mu\nu} + \frac{1}{2}G_i^a G_j^a \overline{U}_{i\mu\nu}\overline{U}_{j\mu\nu}$$
$$\left. - \frac{1}{2}\overline{B}_i^a \overline{B}_j^a V_{i\mu\nu} V_{j\mu\nu} + \frac{1}{2}\overline{G}_i^a \overline{G}_j^a U_{i\mu\nu} U_{j\mu\nu}\right)$$
$$+ \chi_1\left(\overline{V}_{i\mu\nu}\partial^2 V_{i\mu\nu} - \overline{U}_{i\mu\nu}\partial^2 U_{i\mu\nu}\right)$$
$$+ \chi_2\left(\overline{V}_{i\mu\nu}\partial_\mu\partial_\alpha V_{i\nu\alpha} - \overline{U}_{i\mu\nu}\partial_\mu\partial_\alpha U_{i\nu\alpha}\right)$$
$$- \zeta\left(\overline{U}_{i\mu\nu} U_{i\mu\nu}\overline{U}_{j\alpha\beta} U_{j\alpha\beta} + \overline{V}_{i\mu\nu} V_{i\mu\nu}\overline{V}_{j\alpha\beta} V_{j\alpha\beta}\right.$$
$$\left. - 2\overline{U}_{i\mu\nu} U_{i\mu\nu}\overline{V}_{j\alpha\beta} V_{j\alpha\beta}\right) - \Omega_\mu^a D_\mu^{ab}c^b$$
$$+ \frac{g}{2}f^{abc}L^a c^b c^c + gf^{abc}\overline{Y}_i^a c^b B_i^c + gf^{abc}Y_i^a c^b \overline{B}_i^c$$
$$\left. + gf^{abc}\overline{X}_i^a c^b G_i^c + gf^{abc}X_i^a c^b \overline{G}_i^c\right\}. \tag{71}$$

## 3 The complete set of Ward identities

In this section, we have enlisted all known Ward identities, associated to the action (71).

– The Slavnov–Taylor identity

$$\mathcal{S}(\Sigma) \equiv$$
$$\int \mathrm{d}^4x \left[\frac{\delta\Sigma}{\delta\Omega_\mu^a}\frac{\delta\Sigma}{\delta A_\mu^a} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + b^a\frac{\delta\Sigma}{\delta\overline{c}^a} + \omega_I^a\frac{\delta\Sigma}{\delta\phi_I^a}\right.$$
$$\left. + \overline{\phi}_I^a\frac{\delta\Sigma}{\delta\overline{\omega}_I^a} - \overline{M}_{\mu I}^a\frac{\delta\Sigma}{\delta\overline{N}_{\mu I}^a} - N_{\mu I}^a\frac{\delta\Sigma}{\delta M_{\mu I}^a}\right.$$

$$+\left(\frac{\delta\Sigma}{\delta\overline{Y}_i^a}+G_i^a\right)\frac{\delta\Sigma}{\delta B_i^a}+\frac{\delta\Sigma}{\delta Y_i^a}\frac{\delta\Sigma}{\delta\overline{B}_i^a}$$
$$+\frac{\delta\Sigma}{\delta\overline{X}_i^a}\frac{\delta\Sigma}{\delta G_i^a}+\left(\frac{\delta\Sigma}{\delta X_i^a}+\overline{B}_i^a\right)\frac{\delta\Sigma}{\delta\overline{G}_i^a}+\overline{V}_{i\mu\nu}\frac{\delta\Sigma}{\delta\overline{U}_{i\mu\nu}}$$
$$\left.+U_{i\mu\nu}\frac{\delta\Sigma}{\delta V_{i\mu\nu}}-\overline{Y}_i^a\frac{\delta\Sigma}{\delta\overline{X}_i^a}+X_i^a\frac{\delta\Sigma}{\delta Y_i^a}\right]=0\,. \qquad (72)$$

– The Landau gauge fixing

$$\frac{\delta\Sigma}{\delta b^a}=\partial_\mu A_\mu^a\,. \qquad (73)$$

– The antighost equation

$$\frac{\delta\Sigma}{\delta\overline{c}^a}+\partial_\mu\frac{\delta\Sigma}{\delta\Omega_\mu^a}=0\,. \qquad (74)$$

– The ghost equation

$$\mathcal{G}^a(\Sigma)\equiv\int\mathrm{d}^4x\left[\frac{\delta\Sigma}{\delta c^a}+gf^{abc}\left(\overline{c}^b\frac{\delta\Sigma}{\delta b^c}+\phi_I^b\frac{\delta\Sigma}{\delta\omega_I^c}+\overline{\omega}_I^b\frac{\delta\Sigma}{\delta\overline{\phi}_I^c}\right.\right.$$
$$\left.\left.-\overline{N}_{\mu I}^b\frac{\delta\Sigma}{\delta\overline{M}_{\mu I}^c}-M_{\mu I}^b\frac{\delta\Sigma}{\delta N_{\mu I}^c}\right)\right]$$
$$=\Delta_{\mathrm{class}}^a$$
$$=\int\mathrm{d}^4x\,gf^{abc}\left(\Omega_\mu^b A_\mu^c-L^b c^c+\overline{Y}_i^b B_i^c+Y_i^b\overline{B}_i^c\right.$$
$$\left.-\overline{X}_i^b G_i^c-X_i^b\overline{G}_i^c\right)\,. \qquad (75)$$

– The rigid group transformations

$$\mathcal{W}^a(\Sigma)\equiv gf^{abc}\int\mathrm{d}^4x\sum_k\psi_k^b\frac{\delta\Sigma}{\delta\psi_k^c}=0\,,$$
$$\psi_k^a\equiv(A,b,c,\overline{c},\phi,\overline{\phi},\omega,\overline{\omega},B,\overline{B},G,\overline{G},\Omega,L,$$
$$M,\overline{M},N,\overline{N},X,\overline{X},Y,\overline{Y})\,. \qquad (76)$$

– The $SL(2,\mathbb{R})$ invariance [67]

$$\mathcal{D}(\Sigma)\equiv\int\mathrm{d}^4x\left(c^a\frac{\delta\Sigma}{\delta\overline{c}^a}+\frac{\delta\Sigma}{\delta b^a}\frac{\delta\Sigma}{\delta L^a}\right)=0\,. \qquad (77)$$

– The $\overline{\phi}$-equation

$$\frac{\delta\Sigma}{\delta\overline{\phi}_I^a}-\partial_\mu\frac{\delta\Sigma}{\delta\overline{M}_{\mu I}^a}=(1+\chi)\partial_\mu M_{\mu I}^a-gf^{abc}M_{\mu I}^b A_\mu^c\,. \qquad (78)$$

– The $\omega$-equation

$$\frac{\delta\Sigma}{\delta\omega_I^a}+\partial_\mu\frac{\delta\Sigma}{\delta N_{\mu I}^a}+gf^{abc}\overline{\omega}_I^b\frac{\delta\Sigma}{\delta b^c}$$
$$=-(1+\chi)\partial_\mu\overline{N}_{\mu I}^a+gf^{abc}\overline{N}_{\mu I}^b A_\mu^c\,. \qquad (79)$$

– The $\phi$-equation

$$\frac{\delta\Sigma}{\delta\phi_I^a}-\partial_\mu\frac{\delta\Sigma}{\delta M_{\mu I}^a}+gf^{abc}\left(\overline{\phi}_I^b\frac{\delta\Sigma}{\delta b^c}+\overline{\omega}_I^b\frac{\delta\Sigma}{\delta\overline{c}^c}+\overline{N}_{\mu I}^b\frac{\delta\Sigma}{\delta\Omega_\mu^c}\right)$$
$$=(1+\chi)\partial_\mu\overline{M}_{\mu I}^a-gf^{abc}\overline{M}_{\mu I}^b A_\mu^c\,. \qquad (80)$$

– The $\overline{\omega}$-equation

$$\frac{\delta\Sigma}{\delta\overline{\omega}_I^a}+\partial_\mu\frac{\delta\Sigma}{\delta\overline{N}_{\mu I}^a}-gf^{abc}M_{\mu I}^b\frac{\delta\Sigma}{\delta\Omega_\mu^c}$$
$$=(1+\chi)\partial_\mu N_{\mu I}^a-gf^{abc}N_{\mu I}^b A_\mu^c\,. \qquad (81)$$

– The global U($f$) invariance, $f=4(N^2-1)$,

$$\mathcal{Q}_{IJ}(\Sigma)\equiv\int\mathrm{d}^4x\left(\phi_I^a\frac{\delta\Sigma}{\delta\phi_J^a}-\overline{\phi}_J^a\frac{\delta\Sigma}{\delta\overline{\phi}_I^a}+\omega_I^a\frac{\delta\Sigma}{\delta\omega_J^a}\right.$$
$$-\overline{\omega}_J^a\frac{\delta\Sigma}{\delta\overline{\omega}_I^a}+M_{\mu I}^a\frac{\delta\Sigma}{\delta M_J^a}-\overline{M}_J^a\frac{\delta\Sigma}{\delta\overline{M}_I^a}$$
$$\left.+N_{\mu I}^a\frac{\delta\Sigma}{\delta N_{\mu J}^a}-\overline{N}_{\mu J}^a\frac{\delta\Sigma}{\delta\overline{N}_{\mu I}^a}\right)=0\,. \qquad (82)$$

– The rigid symmetry related to the horizon function

$$\mathcal{R}_{IJ}(\Sigma)\equiv\int\mathrm{d}^4x\left(\phi_I^a\frac{\delta\Sigma}{\delta\omega_J^a}-\overline{\omega}_J^a\frac{\delta\Sigma}{\delta\overline{\phi}_I^a}-M_{\mu I}^a\frac{\delta\Sigma}{\delta N_{\mu J}^a}\right.$$
$$\left.+\overline{N}_{\mu J}^a\frac{\delta\Sigma}{\delta\overline{M}_{\mu I}^a}\right)=0\,. \qquad (83)$$

– The symmetries relating the auxiliary fields $\phi,\overline{\phi},\omega,\overline{\omega}$ to the Faddeev–Popov ghost and antighost $c,\overline{c}$

$$\mathcal{W}_I(\Sigma)\equiv\int\mathrm{d}^4x\left(\overline{\omega}_I^a\frac{\delta\Sigma}{\delta\overline{c}^a}-c^a\frac{\delta\Sigma}{\delta\omega_I^a}+\overline{N}_{\mu I}^a\frac{\delta\Sigma}{\delta\Omega_\mu^a}\right)=0\,,$$
$$\mathcal{Q}_I(\Sigma)\equiv\int\mathrm{d}^4x\left(\overline{\phi}_I^a\frac{\delta\Sigma}{\delta\overline{c}^a}+c^a\frac{\delta\Sigma}{\delta\phi_I^a}-M_{\mu I}^a\frac{\delta\Sigma}{\delta\Omega_\mu^a}\right.$$
$$\left.+\frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta\omega_I^a}\right)=0\,. \qquad (84)$$

– The global U(6) invariance

$$\mathcal{Q}_{ij}(\Sigma)\equiv\int\mathrm{d}^4x\left(B_i^a\frac{\delta\Sigma}{\delta B_j^a}-\overline{B}_j^a\frac{\delta\Sigma}{\delta\overline{B}_i^a}+G_i^a\frac{\delta\Sigma}{\delta G_j^a}\right.$$
$$-\overline{G}_j^a\frac{\delta\Sigma}{\delta\overline{G}_i^a}+U_{i\mu\nu}\frac{\delta\Sigma}{\delta U_{j\mu\nu}}-\overline{U}_{j\mu\nu}\frac{\delta\Sigma}{\delta\overline{U}_{i\mu\nu}}$$
$$+V_{i\mu\nu}\frac{\delta\Sigma}{\delta V_{j\mu\nu}}-\overline{V}_{j\mu\nu}\frac{\delta\Sigma}{\delta\overline{V}_{i\mu\nu}}+Y_i^a\frac{\delta\Sigma}{\delta Y_j^a}$$
$$\left.-\overline{Y}_j^a\frac{\delta\Sigma}{\delta\overline{Y}_i^a}+X_i^a\frac{\delta\Sigma}{\delta X_j^a}-\overline{X}_j^a\frac{\delta\Sigma}{\delta\overline{X}_i^a}\right)=0\,. \qquad (85)$$

– The rigid symmetries related to the mass operator

$$\mathcal{R}_{ij}^{(\alpha)}(\Sigma)=0\,,\quad\alpha\in\{1,2,3,4\}$$

with

$$
\mathcal{R}_{ij}^{(1)}(\Sigma) \equiv \int \mathrm{d}^4x \left( B_i^a \frac{\delta\Sigma}{\delta G_j^a} - \overline{G}_j^a \frac{\delta\Sigma}{\delta \overline{B}_i^a} + V_{i\mu\nu} \frac{\delta\Sigma}{\delta U_{j\mu\nu}} \right.
$$
$$
\left. - \overline{U}_{j\mu\nu} \frac{\delta\Sigma}{\delta \overline{V}_{i\mu\nu}} + Y_i^a \frac{\delta\Sigma}{\delta X_j^a} + \overline{X}_j^a \frac{\delta\Sigma}{\delta \overline{Y}_i^a} \right),
$$

$$
\mathcal{R}_{ij}^{(2)}(\Sigma) \equiv \int \mathrm{d}^4x \left( \overline{B}_i^a \frac{\delta\Sigma}{\delta \overline{G}_j^a} + G_j^a \frac{\delta\Sigma}{\delta B_i^a} + \overline{V}_{i\mu\nu} \frac{\delta\Sigma}{\delta \overline{U}_{j\mu\nu}} \right.
$$
$$
\left. + U_{j\mu\nu} \frac{\delta\Sigma}{\delta V_{i\mu\nu}} - \overline{Y}_i^a \frac{\delta\Sigma}{\delta \overline{X}_j^a} - X_j^a \frac{\delta\Sigma}{\delta Y_i^a} \right),
$$

$$
\mathcal{R}_{ij}^{(3)}(\Sigma) \equiv \int \mathrm{d}^4x \left( \overline{B}_i^a \frac{\delta\Sigma}{\delta G_j^a} - \overline{G}_j^a \frac{\delta\Sigma}{\delta B_i^a} - \overline{V}_{i\mu\nu} \frac{\delta\Sigma}{\delta U_{j\mu\nu}} \right.
$$
$$
\left. + \overline{U}_{j\mu\nu} \frac{\delta\Sigma}{\delta V_{i\mu\nu}} + \overline{Y}_i^a \frac{\delta\Sigma}{\delta X_j^a} + \overline{X}_j^a \frac{\delta\Sigma}{\delta Y_i^a} \right),
$$

$$
\mathcal{R}_{ij}^{(4)}(\Sigma) \equiv \int \mathrm{d}^4x \left( B_i^a \frac{\delta\Sigma}{\delta \overline{G}_j^a} + G_j^a \frac{\delta\Sigma}{\delta \overline{B}_i^a} - V_{i\mu\nu} \frac{\delta\Sigma}{\delta \overline{U}_{j\mu\nu}} \right.
$$
$$
\left. - U_{j\mu\nu} \frac{\delta\Sigma}{\delta \overline{V}_{i\mu\nu}} - Y_i^a \frac{\delta\Sigma}{\delta \overline{X}_j^a} + X_j^a \frac{\delta\Sigma}{\delta \overline{Y}_i^a} \right).
$$

## 4 The linearized operators and (anti-)commutation relations

In order to facilitate the upcoming vast amount of algebra required for the determination of the most general counterterm, we shall give her some (anti-)commutation relations between several (linearized) symmetry operators.

Equations (72), (77) and (84) generate, respectively, the following linearized operators:

$$
\mathcal{B}_\Sigma \equiv \int \mathrm{d}^4x \left[ \frac{\delta\Sigma}{\delta\Omega_\mu^a} \frac{\delta}{\delta A_\mu^a} + \frac{\delta\Sigma}{\delta A_\mu^a} \frac{\delta}{\delta\Omega_\mu^a} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a} \frac{\delta}{\delta L^a} \right.
$$
$$
+ b^a \frac{\delta}{\delta\overline{c}^a} + \omega_I^a \frac{\delta}{\delta\phi_I^a} + \overline{\phi}_I^a \frac{\delta}{\delta\overline{\omega}_I^a} - \overline{M}_{\mu I}^a \frac{\delta}{\delta\overline{N}_{\mu I}^a} - N_{\mu I}^a \frac{\delta}{\delta M_{\mu I}^a}
$$
$$
+ \left( \frac{\delta\Sigma}{\delta\overline{Y}_i^a} + G_i^a \right) \frac{\delta}{\delta B_i^a} + \frac{\delta\Sigma}{\delta B_i^a} \frac{\delta}{\delta\overline{Y}_i^a} + \frac{\delta\Sigma}{\delta Y_i^a} \frac{\delta}{\delta\overline{B}_i^a}
$$
$$
+ \frac{\delta\Sigma}{\delta\overline{B}_i^a} \frac{\delta}{\delta Y_i^a} + \frac{\delta\Sigma}{\delta\overline{X}_i^a} \frac{\delta}{\delta G_i^a} + \frac{\delta\Sigma}{\delta G_i^a} \frac{\delta}{\delta\overline{X}_i^a}
$$
$$
+ \left( \frac{\delta\Sigma}{\delta X_i^a} + \overline{B}_i^a \right) \frac{\delta}{\delta\overline{G}_i^a} + \frac{\delta\Sigma}{\delta\overline{G}_i^a} \frac{\delta}{\delta X_i^a} + V_{i\mu\nu} \frac{\delta}{\delta\overline{U}_{i\mu\nu}}
$$
$$
+ U_{i\mu\nu} \frac{\delta}{\delta V_{i\mu\nu}} - \overline{Y}_i^a \frac{\delta}{\delta\overline{X}_i^a} + X_i^a \frac{\delta}{\delta Y_i^a} \right], \tag{86}
$$

$$
\mathcal{D}_\Sigma \equiv \int \mathrm{d}^4x \left( c^a \frac{\delta}{\delta\overline{c}^a} + \frac{\delta\Sigma}{\delta b^a} \frac{\delta}{\delta L^a} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta}{\delta b^a} \right), \tag{87}
$$

$$
\mathcal{Q}_I^\Sigma \equiv \int \mathrm{d}^4x \left( \overline{\phi}_I^a \frac{\delta}{\delta\overline{c}^a} + c^a \frac{\delta}{\delta\phi_I^a} - M_{\mu I}^a \frac{\delta}{\delta\Omega_\mu^a} \right.
$$
$$
\left. + \frac{\delta\Sigma}{\delta L^a} \frac{\delta}{\delta\omega_I^a} + \frac{\delta\Sigma}{\delta\omega_I^a} \frac{\delta}{\delta L^a} \right) = 0. \tag{88}
$$

Consequently, we are able to derive some useful (anti-)commutations relations:

$$
\left[ \frac{\delta}{\delta b^a}, \mathcal{B}_\Sigma \right] = \frac{\delta}{\delta\overline{c}^a} + \partial_\mu \frac{\delta}{\delta\Omega_\mu^a},
$$

$$
\left\{ \mathcal{G}^a, \mathcal{B}_\Sigma \right\} = \mathcal{W}^a,
$$

$$
\left[ \mathcal{D}_\Sigma, \mathcal{B}_\Sigma \right] = 0,
$$

$$
\left[ \frac{\delta}{\delta\overline{\phi}_I^a} - \partial_\mu \frac{\delta}{\delta\overline{M}_{\mu I}^a}, \mathcal{B}_\Sigma \right] = \frac{\delta}{\delta\overline{\omega}_I^a} + \partial_\mu \frac{\delta}{\delta\overline{N}_{\mu I}^a}
$$
$$
- g f^{abc} M_{\mu I}^b \frac{\delta}{\delta\Omega_\mu^c},
$$

$$
\left\{ \frac{\delta}{\delta\omega_I^a} + \partial_\mu \frac{\delta}{\delta N_{\mu I}^a} + g f^{abc} \overline{\omega}_I^b \frac{\delta}{\delta b^c}, \mathcal{B}_\Sigma \right\}
$$
$$
= \frac{\delta\Sigma}{\delta\phi_I^a} - \partial_\mu \frac{\delta\Sigma}{\delta M_{\mu I}^a}
$$
$$
+ g f^{abc} \left( \overline{\phi}_I^b \frac{\delta\Sigma}{\delta b^c} + \overline{\omega}_I^b \frac{\delta\Sigma}{\delta\overline{c}^c} + \overline{N}_{\mu I}^b \frac{\delta\Sigma}{\delta\Omega_\mu^c} \right),
$$

$$
\left\{ \mathcal{R}_{IJ}, \mathcal{B}_\Sigma \right\} = \mathcal{Q}_{IJ},
$$

$$
\left[ \mathcal{W}_I, \mathcal{B}_\Sigma \right] = -\mathcal{Q}_I^\Sigma,
$$

$$
\left\{ \mathcal{R}_{ij}^{(1)}, \mathcal{B}_\Sigma \right\} = \mathcal{Q}_{ij},
$$

$$
\left\{ \mathcal{R}_{ij}^{(2)}, \mathcal{B}_\Sigma \right\} = 0,
$$

$$
\left\{ \mathcal{R}_{ij}^{(3)}, \mathcal{B}_\Sigma \right\} = \int \mathrm{d}^4x \, (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})
$$
$$
\times \left( \overline{B}_k^a \frac{\delta}{\delta B_l^a} - \overline{V}_{k\mu\nu} \frac{\delta}{\delta V_{l\mu\nu}} + \overline{Y}_k^a \frac{\delta}{\delta Y_l^a} \right),
$$

$$
\left\{ \mathcal{R}_{ij}^{(4)}, \mathcal{B}_\Sigma \right\} = \int \mathrm{d}^4x \, (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})
$$
$$
\times \left( G_k^a \frac{\delta}{\delta \overline{G}_l^a} - U_{k\mu\nu} \frac{\delta}{\delta \overline{U}_{l\mu\nu}} - X_k^a \frac{\delta}{\delta \overline{X}_l^a} \right),
$$

$$
\left\{ \mathcal{R}_{ik}^{(1)}, \mathcal{R}_{kj}^{(3)} \right\} = -\int \mathrm{d}^4x \, (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})
$$
$$
\times \left( \overline{G}_k^a \frac{\delta}{\delta G_l^a} - \overline{U}_{k\mu\nu} \frac{\delta}{\delta U_{l\mu\nu}} - \overline{X}_k^a \frac{\delta}{\delta X_l^a} \right),
$$

$$
\left\{ \mathcal{R}_{ik}^{(1)}, \mathcal{R}_{kj}^{(4)} \right\} = -\int \mathrm{d}^4x \, (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})
$$
$$
\times \left( B_k^a \frac{\delta}{\delta \overline{B}_l^a} - V_{k\mu\nu} \frac{\delta}{\delta \overline{V}_{l\mu\nu}} + Y_k^a \frac{\delta}{\delta \overline{Y}_l^a} \right). \tag{89}
$$

## 5 Characterization of the most general counterterm

In order to characterize the most general invariant counterterm, which can be freely added to all orders of perturbation theory, we perturb the classical action $\Sigma$ by adding an arbitrary integrated local polynomial $\Sigma_{\mathrm{CT}}$ in the fields and external sources of dimension bounded by 4 and with

ghost number 0, and we require that the perturbed action $(\Sigma + \eta \Sigma_{\mathrm{CT}})$ satisfies the same Ward identities as $\Sigma$ to the first order in the perturbation parameter $\eta$. Making use of the BRST cohomological results [66], we may write that

$$\Sigma_{\mathrm{CT}} = a_0 \, S_{\mathrm{YM}} + \mathcal{B}_\Sigma \Delta^{(-1)} \,, \tag{90}$$

where $\mathcal{B}_\Sigma$ is the nilpotent linearized Slavnov–Taylor operator of (86),

$$\mathcal{B}_\Sigma \mathcal{B}_\Sigma = 0 \,. \tag{91}$$

The expression $\Delta^{(-1)}$ is an integrated polynomial of ghost number $-1$, in the present case given by

$$
\begin{aligned}
\Delta^{(-1)} = \int \mathrm{d}^4 x \, \Big\{ & a_1 \left( \Omega_\mu^a + \partial_\mu \overline{c}^a \right) A_\mu^a + a_2 \, L^a c^a \\
& + a_3 \left( \overline{Y}_i^a B_i^a - \overline{X}_i^a G_i^a + Y_i^a \overline{B}_i^a - X_i^a \overline{G}_i^a \right) \\
& + a_4 \, \overline{N}_{\mu I}^a \, \partial_\mu \phi_I + a_5 \, M_{\mu I}^a \, \partial_\mu \overline{\omega}_I^a + a_6 \, \overline{U}_{i\mu\nu} \left( \partial_\mu A_\nu^a \right) B_i^a \\
& + a_7 \, \overline{U}_{i\mu\nu} A_\nu^a \, \partial_\mu B_i^a + a_8 \, V_{i\mu\nu} \left( \partial_\mu A_\nu^a \right) \overline{G}_i^a \\
& + a_9 \, V_{i\mu\nu} A_\nu^a \, \partial_\mu \overline{G}_i^a + a_{10} \, \overline{\omega}_I^a \partial^2 \phi_I^a + a_{11} \, \overline{G}_i^a \partial^2 B_i^a \\
& + a_{12} \, \zeta \, \overline{U}_{i\mu\nu} V_{i\mu\nu} \left( \overline{V}_{j\alpha\beta} V_{j\alpha\beta} - \overline{U}_{j\alpha\beta} U_{j\alpha\beta} \right) \\
& + a_{13} \, \chi_1 \, \overline{U}_{i\mu\nu} \partial^2 V_{i\mu\nu} + a_{14} \, \chi_2 \, \overline{U}_{i\mu\nu} \partial_\mu \partial_\alpha V_{i\nu\alpha} \\
& + a_{15} \, \overline{\omega}_I^a \phi_I^a \left( \overline{V}_{i\mu\nu} V_{i\mu\nu} - \overline{U}_{i\mu\nu} U_{i\mu\nu} \right) \\
& + a_{16} \, \lambda_1 \, \overline{G}_i^a B_i^a \left( \overline{V}_{j\mu\nu} V_{j\mu\nu} - \overline{U}_{j\mu\nu} U_{j\mu\nu} \right) \\
& + a_{17} \, \lambda_3 \left( \overline{G}_i^a G_j^a V_{i\mu\nu} \overline{U}_{j\mu\nu} + \overline{G}_i^a B_j^a V_{i\mu\nu} \overline{V}_{j\mu\nu} \right. \\
& \left. - \frac{1}{2} B_i^a B_j^a \overline{U}_{i\mu\nu} \overline{V}_{j\mu\nu} + \frac{1}{2} B_i^a G_j^a \overline{U}_{i\mu\nu} \overline{U}_{j\mu\nu} \right. \\
& \left. - \frac{1}{2} \overline{G}_i^a \overline{B}_j^a V_{i\mu\nu} V_{j\mu\nu} + \frac{1}{2} \overline{G}_i^a \overline{G}_j^a V_{i\mu\nu} U_{j\mu\nu} \right) \\
& + a_{18} \, \frac{\lambda^{abcd} + \mathcal{N}^{abcd}}{16} \, \overline{G}_i^a B_i^b \left( \overline{B}_j^c B_j^d - \overline{G}_j^c G_j^d \right) \\
& + a_{19} \, \chi \, \overline{N}_{\mu I}^a M_{\mu I}^a + a_{20} \, g f^{abc} \overline{N}_{\mu I}^a \, \phi_I^b A_\mu^c \\
& + a_{21} \, g f^{abc} M_{\mu I}^a \, \overline{\omega}_I^b A_\mu^c + a_{22} \, g f^{abc} \overline{\omega}_I^a A_\mu^c \, \partial_\mu \phi_I^b \\
& + a_{23} \, g f^{abc} \overline{\omega}_I^a (\partial_\mu A_\mu^c) \phi_I^b + a_{24} \, g f^{abc} \overline{G}_i^a A_\mu^c \, \partial_\mu B_i^b \\
& + a_{25} \, g f^{abc} \overline{G}_i^a (\partial_\mu A_\mu^c) B_i^b + a_{26} \, g f^{abc} \overline{U}_{i\mu\nu} A_\mu^b A_\nu^c B_i^a \\
& + a_{27} \, g f^{abc} V_{i\mu\nu} A_\mu^b A_\nu^c \overline{G}_i^a + \tilde{\lambda}^{abcd} \overline{G}_i^a B_i^b A_\mu^c A_\mu^d \Big\} \,.
\end{aligned}
\tag{92}
$$

Due to the Ward identities given in Sect. 4, the counterterm $\Sigma_{\mathrm{CT}}$ must obey the following constraints

$$
\begin{aligned}
& \mathcal{B}_\Sigma \Sigma_{\mathrm{CT}} = 0 \,, \quad \mathcal{D}_\Sigma \Sigma_{\mathrm{CT}} = 0 \,, \quad \mathcal{G}^a \Sigma_{\mathrm{CT}} = 0 \,, \\
& \mathcal{W}^a \Sigma_{\mathrm{CT}} = 0 \,, \quad \mathcal{Q}_{IJ} \Sigma_{\mathrm{CT}} = 0 \,, \quad \mathcal{R}_{IJ} \Sigma_{\mathrm{CT}} = 0 \,, \\
& \mathcal{W}_I \Sigma_{\mathrm{CT}} = 0 \,, \quad \mathcal{Q}_I^\Sigma \Sigma_{\mathrm{CT}} = 0 \,, \quad \mathcal{Q}_{ij} \Sigma_{\mathrm{CT}} = 0 \,, \\
& \mathcal{R}_{ij}^{(1,2,3,4)} \Sigma_{\mathrm{CT}} = 0 \,, \quad \left\{ \mathcal{R}_{ij}^{(3,4)}, \mathcal{B}_\Sigma \right\} \Sigma_{\mathrm{CT}} = 0 \,, \\
& \left\{ \mathcal{R}_{ik}^{(1)}, \mathcal{R}_{kj}^{(3,4)} \right\} \Sigma_{\mathrm{CT}} = 0 \,, \quad \frac{\delta \Sigma_{\mathrm{CT}}}{\delta b^a} = 0 \,, \\
& \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \overline{c}^a} + \partial_\mu \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \Omega_\mu^a} = 0 \,, \quad \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \overline{\phi}_I} - \partial_\mu \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \overline{M}_{\mu I}^a} = 0 \,,
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \omega_I^a} + \partial_\mu \frac{\delta \Sigma_{\mathrm{CT}}}{\delta N_{\mu I}^a} = 0 \,, \\
& \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \phi_I^a} - \partial_\mu \frac{\delta \Sigma_{\mathrm{CT}}}{\delta M_{\mu I}^a} + g f^{abc} \left( \overline{\omega}_I^b \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \overline{c}^c} + \overline{N}_{\mu I}^b \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \Omega_\mu^c} \right) = 0 \,, \\
& \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \overline{\omega}_I^a} + \partial_\mu \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \overline{N}_{\mu I}^a} - g f^{abc} M_{\mu I}^b \frac{\delta \Sigma_{\mathrm{CT}}}{\delta \Omega_\mu^c} = 0 \,. \tag{93}
\end{aligned}
$$

By applying the constraints (93), one can show that

$$\tilde{\lambda}^{abcd} = a_{11} \, g^2 f^{ace} f^{edb} \,, \tag{94}$$

and

$$
\begin{aligned}
\Sigma_{\mathrm{CT}} = a_0 \, S_{\mathrm{YM}} + \int \mathrm{d}^4 x \, \Big\{ & a_1 \Big[ A_\mu^a \frac{\delta S_{\mathrm{YM}}}{\delta A_\mu^a} + \left( \Omega_\mu^a + \partial_\mu \overline{c}^a \right) \partial_\mu c^a \\
& + \overline{\phi}_I^a \partial^2 \phi_I^a - \overline{\omega}_I^a \partial^2 \omega_I^a - g f^{abc} \overline{\omega}_I^a \partial_\mu \left( \phi_I^b \partial_\mu c^c \right) \\
& + \overline{M}_{\mu I}^a \partial_\mu \phi_I^a + \overline{N}_{\mu I}^a \left( \partial_\mu \omega_I^a + g f^{abc} \phi_I^b \partial_\mu c^c \right) \\
& - N_{\mu I}^a \partial_\mu \overline{\omega}_I^a + M_{\mu I}^a \left( \partial_\mu \overline{\phi}_I^a + g f^{abc} \overline{\omega}_I^b \partial_\mu c^c \right) \\
& - \left( \overline{M}_\mu^a M_\mu^a - \overline{N}_\mu^a N_\mu^a \right) \Big] \\
& + (2a_2 + a_3) \left( \overline{B}_i^a \partial^2 B_i^a - \overline{G}_i^a \partial^2 G_i^a \right) \\
& - (a_1 + 2a_2 + a_3) \, g f^{abc} \Big[ \overline{B}_i^a \left( \partial_\mu A_\mu^b + 2 A_\mu^b \partial_\mu \right) B_i^c \\
& - \overline{G}_i^a \left( \partial_\mu A_\mu^b + 2 A_\mu^b \partial_\mu \right) G_i^c \Big] \\
& + (2a_1 + 2a_2 + a_3) \, g^2 f^{abd} f^{bce} \left( \overline{B}_i^a B_i^c - \overline{G}_i^a G_i^c \right) \\
& \times A_\mu^d A_\mu^e + [(a_1 + a_2 + a_4) \, 2 \partial_\mu A_\nu^a \\
& + (2a_1 + a_2 + a_4) g f^{abc} A_\mu^b A_\nu^c] \\
& \times \left( \overline{U}_{i\mu\nu} G_i^a + V_{i\mu\nu} \overline{B}_i^a + U_{i\mu\nu} \overline{G}_i^a - \overline{V}_{i\mu\nu} B_i^a \right) \\
& + (4a_2 + a_5) \frac{(\lambda + \mathcal{N})^{abcd}}{16} \left( \overline{B}_i^a B_i^b - \overline{G}_i^a G_i^b \right) \\
& \times \left( \overline{B}_j^c B_j^d - \overline{G}_j^c G_j^d \right) \\
& + (2a_2 + a_6) \lambda_1 \left( \overline{B}_i^a B_i^a - \overline{G}_i^a G_i^a \right) \\
& \times \left( \overline{V}_{i\mu\nu} V_{i\mu\nu} - \overline{U}_{i\mu\nu} U_{i\mu\nu} \right) \\
& + (2a_2 + a_7) \lambda_3 \left( \overline{B}_i^a G_j^a V_{i\mu\nu} \overline{U}_{j\mu\nu} + \overline{G}_i^a G_j^a U_{i\mu\nu} \overline{U}_{j\mu\nu} \right. \\
& + \overline{B}_i^a B_j^a V_{i\mu\nu} \overline{V}_{j\mu\nu} - \overline{G}_i^a B_j^a V_{j\mu\nu} U_{i\mu\nu} \\
& - G_i^a B_j^a \overline{U}_{i\mu\nu} \overline{V}_{j\mu\nu} \\
& + \overline{G}_i^a \overline{B}_j^a U_{i\mu\nu} V_{j\mu\nu} - \frac{1}{2} B_i^a B_j^a \overline{V}_{i\mu\nu} \overline{V}_{j\mu\nu} \\
& + \frac{1}{2} G_i^a G_j^a \overline{U}_{i\mu\nu} \overline{U}_{j\mu\nu} \\
& \left. - \frac{1}{2} \overline{B}_i^a \overline{B}_j^a V_{i\mu\nu} V_{j\mu\nu} + \frac{1}{2} \overline{G}_i^a \overline{G}_j^a U_{i\mu\nu} U_{j\mu\nu} \right) \\
& + a_8 \, \zeta \left( \overline{U}_{i\mu\nu} U_{i\mu\nu} \, \overline{U}_{j\alpha\beta} U_{j\alpha\beta} + \overline{V}_{i\mu\nu} V_{i\mu\nu} \overline{V}_{j\alpha\beta} V_{j\alpha\beta} \right. \\
& \left. - 2 \overline{U}_{i\mu\nu} U_{i\mu\nu} \overline{V}_{j\alpha\beta} V_{j\alpha\beta} \right) \\
& + a_9 \, \chi_1 (\overline{V}_{i\mu\nu} \partial^2 V_{i\mu\nu} - \overline{U}_{i\mu\nu} \partial^2 U_{i\mu\nu}) \\
& + a_{10} \, \chi_2 (\overline{V}_{i\mu\nu} \partial_\mu \partial_\alpha V_{i\nu\alpha} - \overline{U}_{i\mu\nu} \partial_\mu \partial_\alpha U_{i\nu\alpha}) \Big\} \,, \tag{95}
\end{aligned}
$$

where we renamed the coefficients $a_n$ as

$$
\begin{aligned}
&a_3 \to a_2, &&a_{18} \to a_5, &&a_{12} \to a_8, \\
&a_{11} \to a_3, &&a_{16} \to a_6, &&a_{13} \to a_9, \\
&a_6 \to -2a_4, &&a_{17} \to a_7, &&a_{14} \to a_{10}.
\end{aligned}
\tag{96}
$$

## 6 Stability of the action at the quantum level and renormalization factors

As a final step, we must show that the most general counterterm $\Sigma_{\mathrm{CT}}$ can be reabsorbed by means of a multiplicative renormalization of the parameters, fields, and sources already present in the starting action $\Sigma$. Taking

$$
\begin{aligned}
\psi_0 &= Z_\psi^{1/2}\, \psi\,, \\
J_0 &= Z_J\, J\,, \\
\xi_0 &= Z_\xi\, \xi\,, \\
\lambda_0^{abcd} &= Z_\lambda\, \lambda^{abcd} + \mathcal{Z}^{abcd}\,,
\end{aligned}
\tag{97}
$$

where

$$
\begin{aligned}
\psi &= \{A, b, c, \overline{c}, \phi, \overline{\phi}, \omega, \overline{\omega}, B, \overline{B}, G, \overline{G}\}\,, \\
J &= \{\Omega, L, M, \overline{M}, N, \overline{N}, U, \overline{U}, V, \overline{V}, X, \overline{X}, Y, \overline{Y}\}\,, \\
\xi &= \{g, \chi, \chi_1, \chi_2, \zeta, \lambda_1, \lambda_3\}\,,
\end{aligned}
\tag{98}
$$

we must show that

$$
\Sigma(\psi_0, J_0, \xi_0) = \Sigma(\psi, J, \xi) + \eta\, \Sigma_{\mathrm{CT}}(\psi, J, \xi) + O(\eta^2)\,. \tag{99}
$$

After some algebra, for the renormalization factors $\{Z\}$ we obtain

$$
\begin{aligned}
Z_b &= Z_A^{-1}\,, \quad Z_c = Z_{\overline{c}} = Z_g^{-1} Z_A^{-1/2}\,, \\
Z_\phi &= Z_{\overline{\phi}} = Z_g^{-1} Z_A^{-1/2}\,, \quad Z_{\overline{\omega}} = Z_g^{-2}\,, \quad Z_\omega = Z_A^{-1}\,, \\
Z_M &= Z_{\overline{M}} = Z_\phi^{1/2} = Z_g^{-1/2} Z_A^{-1/4}\,, \\
Z_{\overline{N}} &= Z_{\overline{\omega}}^{1/2} = Z_g^{-1}\,, \quad Z_N = Z_\omega^{1/2} = Z_A^{-1/2}\,, \\
Z_\Omega &= Z_c^{1/2} = Z_g^{-1/2} Z_A^{-1/4}\,, Z_L = Z_A^{1/2}\,, \\
Z_X &= Z_{\overline{X}} = Z_Y = Z_{\overline{Y}} = Z_g^{-1/2} Z_A^{1/4} Z_B^{-1/2}\,, \\
Z_A &= 1 + \eta(a_0 + 2a_1)\,, \quad Z_g = 1 - \eta\frac{a_0}{2}\,, \\
Z_B &= Z_{\overline{B}} = Z_G = Z_{\overline{G}} = 1 + \eta(2a_2 + a_3)\,, \\
Z_V &= Z_{\overline{V}} = Z_U = Z_{\overline{U}} = 1 - \eta\left(\frac{a_0}{2} + \frac{a_3}{2} - a_4\right)\,, \\
Z_\lambda &= 1 + \eta(a_5 - 2a_3)\,, \quad \mathcal{Z}^{abcd} = \eta(4a_2 + a_5)\mathcal{N}^{abcd}\,, \\
Z_{\lambda_1} &= 1 + \eta(a_0 - 2a_4 + a_6)\,, \\
Z_{\lambda_3} &= 1 + \eta(a_0 - 2a_4 + a_7)\,, \\
Z_\zeta &= 1 + \eta(2a_0 + 2a_3 - 4a_4 - a_8)\,, \\
Z_{\chi_1} &= 1 + \eta(a_0 + a_3 - 2a_4 + a_9)\,, \\
Z_{\chi_2} &= 1 + \eta(a_0 + a_3 - 2a_4 + a_{10})\,,
\end{aligned}
\tag{100}
$$

hereby confirming the renormalizability to all orders of perturbation theory of the action (71). We draw attention to the fact that the renormalization of the quartic

tensor coupling $\lambda^{abcd}$ involves an additional additive part given by $\mathcal{Z}^{abcd}$ [63]. As a consequence, $\lambda^{abcd} = 0$ is a not a fixed point of the model. This originates from the fact that the interactions proportional to the other coupling $g^2$ reintroduce by quantum effects the tensor coupling $\propto \left(\overline{B}_{\mu\nu}^a B_{\mu\nu}^b - \overline{G}_{\mu\nu}^a G_{\mu\nu}^b\right)\left(\overline{B}_{\rho\sigma}^c B_{\rho\sigma}^d - \overline{G}_{\rho\sigma}^c G_{\rho\sigma}^d\right)$. This is nicely reflected in the renormalization group function of $\lambda^{abcd}$, that was calculated at one loop order in [63] using dimensional regularization in $d = 4 - 2\varepsilon$ dimensions and the $\overline{\mathrm{MS}}$ scheme,

$$
\begin{aligned}
\mu\frac{\partial}{\partial\mu}\lambda^{abcd} = {}&-2\varepsilon\lambda^{abcd} + \Bigg[\frac{1}{4}\left(\lambda^{abpq}\lambda^{cpdq} + \lambda^{apbq}\lambda^{cdpq}\right. \\
&+ \left.\lambda^{apcq}\lambda^{bpdq} + \lambda^{apdq}\lambda^{bpcq}\right) \\
&- 12C_A\lambda^{abcd}a + 8C_A f^{abp} f^{cdp} a^2 \\
&+ 16C_A f^{adp} f^{bcp} a^2 + 96 d_A^{abcd} a^2\Bigg]\,, \tag{101}
\end{aligned}
$$

where $a = \frac{g^2}{16\pi^2}$ and $d_A^{abcd}$ is the totally symmetric rank four tensor defined by $d_A^{abcd} = \mathrm{Tr}\left(T_A^a T_A^{(b} T_A^c T_A^{d)}\right)$. Clearly, we have

$$
\mu\frac{\partial}{\partial\mu}\lambda^{abcd} \neq 0\,, \quad \text{for} \quad \lambda^{abcd} = 0\,. \tag{102}
$$

## 7 The physical action and some of its properties

We have shown the renormalizability of the complete action (71). In particular, since renormalizability holds for all possible values of the sources, we have also proven it in the case that the external source part, (65), is zero, while the other sources attain their physical values dictated by (46) and (55), yielding the complete physical action

$$
\begin{aligned}
S_{\mathrm{physical}} = {}&\int \mathrm{d}^4x \left(\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a\right) \\
&+ \int \mathrm{d}^4x \left(b^a \partial_\mu A_\mu^a + \overline{c}^a \partial_\mu D_\mu^{ab} c^b\right) \\
&+ \int \mathrm{d}^4x \left(-\overline{\varphi}_\mu^{ac} \partial_\nu\left(\partial_\nu \varphi_\mu^{ac} + g f^{abm} A_\nu^b \varphi_\mu^{mc}\right)\right. \\
&+ \overline{\omega}_\mu^{ac} \partial_\nu\left(\partial_\nu \omega_\mu^{ac} + g f^{abm} A_\nu^b \omega_\mu^{mc}\right) \\
&+ \left.\overline{\omega}_\mu^{ac} \partial_\nu\left(g f^{abd} \phi_\mu^{bc} D_\nu^{de} c^e\right)\right) \\
&+ \int \mathrm{d}^4x \left[\gamma^2 g f^{abc} A_\mu^b \varphi_\mu^{ac} - \gamma^2 g f^{abc} A_\mu^b \overline{\varphi}_\mu^{ac}\right. \\
&-4\left(N^2 - 1\right)\gamma^4\Big] \\
&+ \int \mathrm{d}^4x \left(\frac{\mathrm{i}m}{4}(B - \overline{B})_{\mu\nu}^a F_{\mu\nu}^a\right. \\
&+ \frac{1}{4}\left(\overline{B}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} B_{\mu\nu}^c - \overline{G}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} G_{\mu\nu}^c\right)\bigg) \\
&+ \int \mathrm{d}^4x \left(-\frac{3}{8}m^2\lambda_1\left(\overline{B}_{\mu\nu}^a B_{\mu\nu}^a - \overline{G}_{\mu\nu}^a G_{\mu\nu}^a\right)\right.
\end{aligned}
$$

$$+ m^2 \frac{\lambda_3}{32} \left( \overline{B}^a_{\mu\nu} - B^a_{\mu\nu} \right)^2 \right)$$
$$+ \int \mathrm{d}^4 x \left( \frac{\lambda^{abcd}}{16} \left( \overline{B}^a_{\mu\nu} B^b_{\mu\nu} - \overline{G}^a_{\mu\nu} G^b_{\mu\nu} \right) \right.$$
$$\left. \times \left( \overline{B}^c_{\rho\sigma} B^d_{\rho\sigma} - \overline{G}^c_{\rho\sigma} G^d_{\rho\sigma} \right) \right) - \int \mathrm{d}^4 x \left( \frac{9}{4} \zeta m^4 \right).$$

(103)

Apparently, the symmetry content of the action (71) given in Sect. 3 is sufficiently powerful to avoid mixing between the Zwanziger fields/sources on one hand and the mass related fields/sources on the other hand.

The term $\propto \zeta m^4$ in the final action (103) is irrelevant for the renormalization of Green functions, but it becomes important when one looks at the renormalization of the vacuum energy $E(m)$. The parameter $\zeta$ is the so-called LCO parameter, and its value ought to be fixed by requiring a homogeneous linear renormalization group equation for $E(m)$, whereby $\zeta$ is made a function of the available couplings. This point is, however, beyond the scope of this paper; the interested reader is kindly referred to e.g. [47, 49, 52, 68] for more details. It is a remarkable feature of the Zwanziger action that there is no need for such a LCO parameter in front of the $\gamma^4$-term in the action (103) [14, 41, 42].

If we make an abstraction of the Gribov–Zwanziger part for the moment, we established a "supersymmetry" for the action $S^{m=0,\gamma=0}_{\mathrm{physical}}$, generated by [63]

$$\delta_s B^a_{\mu\nu} = G^a_{\mu\nu}, \quad \delta_s G^a_{\mu\nu} = 0,$$
$$\delta_s \overline{G}^a_{\mu\nu} = \overline{B}^a_{\mu\nu}, \quad \delta_s \overline{B}^a_{\mu\nu} = 0,$$
$$\delta_s \Psi = 0 \quad \text{for all other fields} \quad \Psi,$$
$$\delta_s^2 = 0,$$
$$\delta_s \left( S^{m=0,\gamma=0}_{\mathrm{physical}} \right) = 0.$$

(104)

We used this symmetry in [63] to show that the massless version of our gauge model is equivalent to Yang–Mills ordinary gauge theories, despite the extra (quartic) interactions between the fields $B^a_{\mu\nu}$, $\overline{B}^a_{\mu\nu}$, $G^a_{\mu\nu}$ and $\overline{G}^a_{\mu\nu}$. A completely similar $\delta_s$-cohomological argument as presented in [63] can be used here to actually prove that the action $S^{m=0}_{\mathrm{physical}}$ and the original Gribov–Zwanziger action give rise to the *same* Green functions at any order of perturbation theory when we restricts ourselves to those functionals built from fields in the original Gribov–Zwanziger action, meaning that the quartic coupling $\lambda^{abcd}$ cancels out from the final results.

The combination of the previous result and the already mentioned absence of mixing, also implies that the already known renormalization group functions and relations for the Gribov–Zwanziger action [14, 41, 42] and massive gauge model [46, 63] remain valid when both are combined into one action, at least whenever massless renormalization schemes like the $\overline{\mathrm{MS}}$ one are employed.

When the sources are set equal to their physical values (46) and (55) in order to obtain the action $S_{\mathrm{physical}}$, the BRST symmetry (33) is however broken. It is worth having a somewhat more detailed look at this.

# 8 The breaking of the Slavnov–Taylor identity scrutinized

## 8.1 The case of the massive gauge model without the Gribov restriction

In order to avoid too lengthy expressions, we shall momentarily skip the Gribov restriction and concentrate on the massive gauge model already studied in earlier papers [46, 63].

Let $\widetilde{\Sigma}$ thus be the complete action given by

$$\widetilde{\Sigma} = S_{\mathrm{YM}} + \int \mathrm{d}^4 x \left\{ b^a \, \partial_\mu A^a_\mu + \overline{c}^a \partial_\mu D^{ab}_\mu c^b \right.$$
$$+ \overline{B}^a_i D^{ab}_\mu D^{bc}_\mu B^c_i - \overline{G}^a_i D^{ab}_\mu D^{bc}_\mu G^c_i$$
$$+ F^a_{\mu\nu} \left( \overline{U}_{i\mu\nu} G^a_i + V_{i\mu\nu} \overline{B}^a_i - \overline{V}_{i\mu\nu} B^a_i + U_{i\mu\nu} \overline{G}^a_i \right)$$
$$+ \lambda_1 \left( \overline{B}^a_i B^a_i - \overline{G}^a_i G^a_i \right) \left( \overline{V}_{j\mu\nu} V_{j\mu\nu} - \overline{U}_{j\mu\nu} U_{j\mu\nu} \right)$$
$$+ \frac{\lambda^{abcd}}{16} \left( \overline{B}^a_i B^b_i - \overline{G}^a_i G^b_i \right) \left( \overline{B}^c_j B^d_j - \overline{G}^c_j G^d_j \right)$$
$$+ \lambda_3 \left( \overline{B}^a_i G^a_j V_{i\mu\nu} \overline{U}_{j\mu\nu} + \overline{G}^a_i G^a_j U_{i\mu\nu} \overline{U}_{j\mu\nu} \right.$$
$$+ \overline{B}^a_i B^a_j V_{i\mu\nu} \overline{V}_{j\mu\nu} - \overline{G}^a_i B^a_j V_{j\mu\nu} U_{i\mu\nu}$$
$$- G^a_i B^a_j \overline{U}_{i\mu\nu} \overline{V}_{j\mu\nu} + \overline{G}^a_i \overline{B}^a_j U_{i\mu\nu} V_{j\mu\nu}$$
$$- \frac{1}{2} B^a_i B^a_j \overline{V}_{i\mu\nu} \overline{V}_{j\mu\nu} + \frac{1}{2} G^a_i G^a_j \overline{U}_{i\mu\nu} \overline{U}_{j\mu\nu}$$
$$\left. - \frac{1}{2} \overline{B}^a_i \overline{B}^a_j V_{i\mu\nu} V_{j\mu\nu} + \frac{1}{2} \overline{G}^a_i \overline{G}^a_j U_{i\mu\nu} U_{j\mu\nu} \right)$$
$$+ \chi_1 \left( \overline{V}_{i\mu\nu} \partial^2 V_{i\mu\nu} - \overline{U}_{i\mu\nu} \partial^2 U_{i\mu\nu} \right)$$
$$+ \chi_2 \left( \overline{V}_{i\mu\nu} \partial_\mu \partial_\alpha V_{i\nu\alpha} - \overline{U}_{i\mu\nu} \partial_\mu \partial_\alpha U_{i\nu\alpha} \right)$$
$$- \zeta \left( \overline{U}_{i\mu\nu} U_{i\mu\nu} \overline{U}_{j\alpha\beta} U_{j\alpha\beta} + \overline{V}_{i\mu\nu} V_{i\mu\nu} \overline{V}_{j\alpha\beta} V_{j\alpha\beta} \right.$$
$$\left. - 2 \overline{U}_{i\mu\nu} U_{i\mu\nu} \overline{V}_{j\alpha\beta} V_{j\alpha\beta} \right) - \Omega^a_\mu D^{ab}_\mu c^b$$
$$+ \frac{g}{2} f^{abc} L^a c^b c^c + g f^{abc} \overline{Y}^a_i c^b B^c_i + g f^{abc} Y^a_i c^b \overline{B}^c_i$$
$$\left. + g f^{abc} \overline{X}^a_i c^b G^c_i + g f^{abc} X^a_i c^b \overline{G}^c_i \right\}.$$

(105)

This action $\widetilde{\Sigma}$ obeys the Slavnov–Taylor identity

$$\widetilde{\mathcal{S}}(\widetilde{\Sigma}) = 0,$$

(106)

with

$$\widetilde{\mathcal{S}}(\widetilde{\Sigma}) = \int \mathrm{d}^4 x \left[ \frac{\delta \widetilde{\Sigma}}{\delta \Omega^a_\mu} \frac{\delta \widetilde{\Sigma}}{\delta A^a_\mu} + \frac{\delta \widetilde{\Sigma}}{\delta L^a} \frac{\delta \widetilde{\Sigma}}{\delta c^a} + b^a \frac{\delta \widetilde{\Sigma}}{\delta \overline{c}^a} \right.$$
$$+ \left( \frac{\delta \widetilde{\Sigma}}{\delta \overline{Y}^a_i} + G^a_i \right) \frac{\delta \widetilde{\Sigma}}{\delta B^a_i} + \frac{\delta \widetilde{\Sigma}}{\delta Y^a_i} \frac{\delta \widetilde{\Sigma}}{\delta \overline{B}^a_i} + \frac{\delta \widetilde{\Sigma}}{\delta \overline{X}^a_i} \frac{\delta \widetilde{\Sigma}}{\delta G^a_i}$$
$$+ \left( \frac{\delta \widetilde{\Sigma}}{\delta X^a_i} + \overline{B}^a_i \right) \frac{\delta \widetilde{\Sigma}}{\delta \overline{G}^a_i} + \overline{V}_{i\mu\nu} \frac{\delta \widetilde{\Sigma}}{\delta \overline{U}_{i\mu\nu}} + U_{i\mu\nu} \frac{\delta \widetilde{\Sigma}}{\delta V_{i\mu\nu}}$$
$$\left. - \overline{Y}^a_i \frac{\delta \widetilde{\Sigma}}{\delta \overline{X}^a_i} + X^a_i \frac{\delta \widetilde{\Sigma}}{\delta Y^a_i} \right].$$

(107)

Since the theory is stable and free from anomalies at the quantum level, we may write down a renormalized 1PI

quantum vertex functional [66],

$$\widetilde{\Gamma} = \widetilde{\Sigma} + \hbar \widetilde{\Gamma}^{,(1)} + \dots , \qquad (108)$$

which fulfills the quantum version of the Slavnov–Taylor identity (106), i.e.

$$\widetilde{\mathcal{S}}(\widetilde{\Gamma}) = 0 , \qquad (109)$$

$$
\widetilde{\mathcal{S}}(\widetilde{\Gamma}) = \int \mathrm{d}^4 x \left[ \frac{\delta\widetilde{\Gamma}}{\delta\Omega_\mu^a} \frac{\delta\widetilde{\Gamma}}{\delta A_\mu^a} + \frac{\delta\widetilde{\Gamma}}{\delta L^a} \frac{\delta\widetilde{\Gamma}}{\delta c^a} + b^a \frac{\delta\widetilde{\Gamma}}{\delta\overline{c}^a} \right.
$$
$$
+ \left( \frac{\delta\widetilde{\Gamma}}{\delta\overline{Y}_i^a} + G_i^a \right) \frac{\delta\widetilde{\Gamma}}{\delta B_i^a} + \frac{\delta\widetilde{\Gamma}}{\delta Y_i^a} \frac{\delta\widetilde{\Gamma}}{\delta\overline{B}_i^a} + \frac{\delta\widetilde{\Gamma}}{\delta\overline{X}_i^a} \frac{\delta\widetilde{\Gamma}}{\delta G_i^a}
$$
$$
+ \left( \frac{\delta\widetilde{\Gamma}}{\delta X_i^a} + \overline{B}_i^a \right) \frac{\delta\widetilde{\Gamma}}{\delta\overline{G}_i^a} + \overline{V}_{i\mu\nu} \frac{\delta\widetilde{\Gamma}}{\delta\overline{U}_{i\mu\nu}} + U_{i\mu\nu} \frac{\delta\widetilde{\Gamma}}{\delta V_{i\mu\nu}}
$$
$$
\left. - \overline{Y}_i^a \frac{\delta\widetilde{\Gamma}}{\delta\overline{X}_i^a} + X_i^a \frac{\delta\widetilde{\Gamma}}{\delta Y_i^a} \right] . \qquad (110)
$$

Let us now analyze the quantum properties of the model when the sources attain their physical values (55). First of all, let us give a look at the classical action $\widetilde{\Sigma}_{ph}$, obtained by $\widetilde{\Sigma}$ by setting the sources to their physical values, namely

$$\widetilde{\Sigma}_{\mathrm{ph}} = \widetilde{\Sigma}\Big|_{\text{physical value of } \left( V_{i\mu\nu}, \overline{V}_{i\mu\nu}, U_{i\mu\nu}, \overline{U}_{i\mu\nu} \right)} , \qquad (111)$$

or explicitly

$$
\widetilde{\Sigma}_{\mathrm{ph}} = \int \mathrm{d}^4 x \left( \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \right)
$$
$$
+ \int \mathrm{d}^4 x \; (b^a \partial_\mu A_\mu^a + \overline{c}^a \partial_\mu D_\mu^{ab} c^b)
$$
$$
+ \int \mathrm{d}^4 x \left( \frac{\mathrm{i}m}{4} (B - \overline{B})_{\mu\nu}^a F_{\mu\nu}^a \right.
$$
$$
+ \frac{1}{4} \left( \overline{B}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} B_{\mu\nu}^c - \overline{G}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} G_{\mu\nu}^c \right) \Big)
$$
$$
+ \int \mathrm{d}^4 x \left( -\frac{3}{8} m^2 \lambda_1 \left( \overline{B}_{\mu\nu}^a B_{\mu\nu}^a - \overline{G}_{\mu\nu}^a G_{\mu\nu}^a \right) \right.
$$
$$
+ m^2 \frac{\lambda_3}{32} \left( \overline{B}_{\mu\nu}^a - B_{\mu\nu}^a \right)^2 \Big)
$$
$$
+ \int \mathrm{d}^4 x \left( \frac{\lambda^{abcd}}{16} \left( \overline{B}_{\mu\nu}^a B_{\mu\nu}^b - \overline{G}_{\mu\nu}^a G_{\mu\nu}^b \right) \right.
$$
$$
\left. \times \left( \overline{B}_{\rho\sigma}^c B_{\rho\sigma}^d - \overline{G}_{\rho\sigma}^c G_{\rho\sigma}^d \right) \right) - \int \mathrm{d}^4 x \left( \frac{9}{4} \zeta m^4 \right) .
$$
$$
\qquad (112)
$$

It is easy to check that $\widetilde{\Sigma}_{\mathrm{ph}}$ is not BRST invariant with respect to (33). In fact, it turns out that

$$
s\widetilde{\Sigma}_{\mathrm{ph}} = \frac{\mathrm{i}m}{4} \int \mathrm{d}^4 x G_{\mu\nu}^a F_{\mu\nu}^a
$$
$$
- \lambda_3 \frac{m^2}{16} \int \mathrm{d}^4 x \left( \overline{B}_{\mu\nu}^a - B_{\mu\nu}^a \right) G_{\mu\nu}^a . \quad (113)
$$

This equation shows that the breaking of the BRST symmetry (33) is not linear in the quantum fields, and hence the breaking terms have to be treated as composite operators [66]. Therefore, (113) cannot be renormalized as it stands. The two breaking terms have to be taken into proper account. This is precisely achieved by introducing the local sources $\left( V_{i\mu\nu}, \overline{V}_{i\mu\nu}, U_{i\mu\nu}, \overline{U}_{i\mu\nu} \right)$. In other words, these sources allow us to take into account e.g. the presence of $\int \mathrm{d}^4 x G_{\mu\nu}^a F_{\mu\nu}^a$ and its renormalization, which is expressed by the renormalization factor of the source $\overline{U}_{i\mu\nu}$.

We would like to understand what happens to the BRST symmetry at the quantum level, when the sources attain their physical values. It is instructive to study this limit by means of the Slavnov–Taylor identity (109). Let $\widetilde{\Gamma}_{\mathrm{ph}}$ be the 1PI functional obtained from $\widetilde{\Gamma}$ when the sources $\left( V_{i\mu\nu}, \overline{V}_{i\mu\nu}, U_{i\mu\nu}, \overline{U}_{i\mu\nu} \right)$ attain their physical values

$$\widetilde{\Gamma}_{\mathrm{ph}} = \widetilde{\Gamma}\Big|_{\text{physical value of } \left( V_{i\mu\nu}, \overline{V}_{i\mu\nu}, U_{i\mu\nu}, \overline{U}_{i\mu\nu} \right)} . \qquad (114)$$

We can write

$$
\int \mathrm{d}^4 x \overline{V}_{i\mu\nu} \frac{\delta\widetilde{\Gamma}}{\delta\overline{U}_{i\mu\nu}} \Bigg|_{\text{physical value}}
$$
$$
= -\frac{\mathrm{i}m}{4} \left[ \int \mathrm{d}^4 x G_{\mu\nu}^a F_{\mu\nu}^a \cdot \widetilde{\Gamma} \right]_{\text{physical value}}
$$
$$
+ \lambda_3 \frac{m^2}{16} \left[ \int \mathrm{d}^4 x \left( \overline{B}_{\mu\nu}^a - B_{\mu\nu}^a \right) G_{\mu\nu}^a \cdot \widetilde{\Gamma} \right]_{\text{physical value}}
$$
$$
\qquad (115)
$$

where e.g. $\left[ \left( \int \mathrm{d}^4 x G_{\mu\nu}^a F_{\mu\nu}^a \right) \cdot \widetilde{\Gamma} \right]$ stands for the generator of the 1PI Green functions with the insertion of the composite operator $\left( \int \mathrm{d}^4 x G_{\mu\nu}^a F_{\mu\nu}^a \right)$. Of course, it holds that $\left[ \dots \cdot \widetilde{\Gamma} \right]_{\text{physical value}} = \left[ \dots \cdot \widetilde{\Gamma}_{\mathrm{ph}} \right]$. It follows that the quantum action $\widetilde{\Gamma}_{\mathrm{ph}}$ obeys the broken Slavnov–Taylor identity

$$
\widetilde{\mathcal{S}}(\widetilde{\Gamma}_{\mathrm{ph}}) = \frac{\mathrm{i}m}{4} \left[ \int \mathrm{d}^4 x G_{\mu\nu}^a F_{\mu\nu}^a \cdot \widetilde{\Gamma} \right]_{\text{physical value}}
$$
$$
- \lambda_3 \frac{m^2}{16} \left[ \int \mathrm{d}^4 x \left( \overline{B}_{\mu\nu}^a - B_{\mu\nu}^a \right) G_{\mu\nu}^a \cdot \widetilde{\Gamma} \right]_{\text{physical value}}
$$
$$
\qquad (116)
$$

where

$$
\widetilde{\mathcal{S}}(\widetilde{\Gamma}_{\mathrm{ph}}) = \int \mathrm{d}^4 x \left[ \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta\Omega_\mu^a} \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta A_\mu^a} + \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta L^a} \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta c^a} + b^a \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta\overline{c}^a} \right.
$$
$$
+ \left( \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta\overline{Y}_i^a} + G_i^a \right) \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta B_i^a} + \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta Y_i^a} \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta\overline{B}_i^a}
$$
$$
+ \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta\overline{X}_i^a} \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta G_i^a} + \left( \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta X_i^a} + \overline{B}_i^a \right) \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta\overline{G}_i^a}
$$
$$
\left. - \overline{Y}_i^a \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta\overline{X}_i^a} + X_i^a \frac{\delta\widetilde{\Gamma}_{\mathrm{ph}}}{\delta Y_i^a} \right] . \qquad (117)
$$

It is worth underlining here that (116) is in fact nothing more than a direct consequence of the Slavnov–Taylor identity (109), when the local sources $\left(V_{i\mu\nu}, \overline{V}_{i\mu\nu}, U_{i\mu\nu}, \overline{U}_{i\mu\nu}\right)$ attain their physical values (55).

We conclude that $\Gamma_{\mathrm{ph}}$ does not obey an exact Slavnov–Taylor identity. Of course, (116) translates at the quantum level the fact that the classical action $\widetilde{\Sigma}_{\mathrm{ph}}$, obtained from $\widetilde{\Sigma}$ by bringing the sources to their physical values, is not BRST invariant, according to (113). However, even if $\Gamma_{\mathrm{ph}}$ does not obey an exact Slavnov–Taylor identity, (116) has far reaching consequences on the behavior of the 1PI Green functions obtained from $\Gamma_{\mathrm{ph}}$, i.e. when the sources are set to their physical values.

Let us consider the breaking term $\left[\left(\int \mathrm{d}^4 x G_{\mu\nu}^a F_{\mu\nu}^a\right) \cdot \Gamma\right]$. Typically, Slavnov–Taylor identities at the level of Green functions are obtained by acting with a test operator like $\frac{\delta^n}{\delta\varphi(x_1)\ldots\delta\varphi(x_n)}$, with $\varphi$ any generic field, on expression (116), and setting all sources and fields equal to zero at the end. The condition to be fulfilled so that the breaking would be harmless is quite easily found, since the breaking term will vanish whenever

$$\frac{\delta^n \left[\left(\int \mathrm{d}^4 x G_{\mu\nu}^a F_{\mu\nu}^a\right) \cdot \Gamma\right]}{\delta\phi(x_1)\ldots\delta\phi(x_n)} \tag{118}$$
$$= \left\langle \left(\int \mathrm{d}^4 x G_{\mu\nu}^a(x) F_{\mu\nu}^a(x)\right) \varphi(x_1)\ldots\varphi(x_n) \right\rangle_{\mathrm{1PI}} = 0\,,$$

meaning that the 1PI Green function with the insertion of the operator $\left(\int \mathrm{d}^4 x G_{\mu\nu}^a F_{\mu\nu}^a\right)$ and with $n$ amputated external $\phi$-legs should vanish. Thus, if the condition (118) holds and an analogous one for the other breaking term, the right hand side of (116) is harmless, so that everything goes as if the theory would fulfill an unbroken Slavnov–Taylor identity, namely

$$\widetilde{\mathcal{S}}(\widetilde{\Gamma}_{\mathrm{ph}}) = 0\,. \tag{119}$$

The set of identities for which this happens is quite large. Certainly, it contains all Slavnov–Taylor identities that are obtained from (116) by acting only on the original Yang–Mills fields or even the $B_{\mu\nu}^a$ and $\overline{B}_{\mu\nu}^a$ fields. In this case, there is no way to obtain a nonvanishing contribution to the breaking term because of the presence of the $G_{\mu\nu}^a$-ghost field in the right hand side of (118). For example, the Slavnov–Taylor identity for the 1PI gluon propagator can be obtained from (116) by acting on it with the test operator $\frac{\delta^2}{\delta c(x)\delta A_\nu(y)}$ and setting all fields and other external sources equal to zero. The breaking terms will be irrelevant as the Green function $\left\langle \left(\int \mathrm{d}^4 z G_{\mu\nu}^a(z) F_{\mu\nu}^a(z)\right) c(x) A_\nu(y)\right\rangle$ as well as the other one are trivially zero.

We conclude that most Green functions will behave as if the theory obeys the unbroken Slavnov–Taylor identity (119). The same considerations outlined for the Slavnov–Taylor identity can be repeated for the other Ward identities. The corresponding breaking terms will always contain the integrated ghost fields $G_{\mu\nu}^a$ and/or $\overline{G}_{\mu\nu}^a$, which, in most cases, will lead to vanishing contributions when inserted into a Green function, thereby making the breaking harmless.

The beauty in all this is exactly the fact that the breaking of the Slavnov–Taylor and other Ward identities can be brought under control at the quantum level by the introduction of a suitable set of local sources. All the renormalization results of the action with arbitrary values of the sources are then preserved once the sources are put equal to specific values.

Since the classical part of the action (112), obtained by skipping the gauge fixing term (10), is gauge invariant with respect to the gauge transformations (53), we expect that there should be a nilpotent BRST generator at the quantum level for the gauge fixed action (112). Nevertheless, we have just seen that the BRST operator (33) no longer generates an exact symmetry of the action (112). As was already discussed in [63], the nilpotent transformation

$$s' A_\mu^a = -D_\mu^{ab} c^b\,,$$
$$s' c^a = \frac{g}{2} f^{abc} c^a c^b\,,$$
$$s' B_{\mu\nu}^a = g f^{abc} c^b B_{\mu\nu}^c\,,$$
$$s' \overline{B}_{\mu\nu}^a = g f^{abc} c^b \overline{B}_{\mu\nu}^c\,,$$
$$s' G_{\mu\nu}^a = g f^{abc} c^b G_{\mu\nu}^c\,,$$
$$s' \overline{G}_{\mu\nu}^a = g f^{abc} c^b \overline{G}_{\mu\nu}^c\,,$$
$$s' \overline{c}^a = b^a\,,$$
$$s' b^a = 0\,,$$
$$s'^2 = 0 \tag{120}$$

generates an invariance of (112). One shall easily recognize that there is an intimate connection between the transformations $s$ (33), $s'$ (120) and $\delta_s$ (104), namely we have

$$s = s' + \delta_s\,. \tag{121}$$

Then we can say that the breaking of the BRST symmetry $s$ and its associated Slavnov–Taylor identity is entirely due to the loss of the supersymmetry $\delta_s$ when the physical limit (55) is taken [63].

Evidently, the unbroken BRST symmetry $s'$ can be used to construct unbroken Slavnov–Taylor identities between the Green functions of the massive gauge model (112).

## 8.2 The case of the massive gauge model with Gribov restriction

A very similar analysis can be made when we consider the full action (103). In this case, there are additional breaking terms coming from the physical limit (46) of the Zwanziger sources. In particular, applying the transformations $s$ or $s'$ on (103), we find

$$sS_{\mathrm{physical}} = \frac{\mathrm{i}m}{4} \int \mathrm{d}^4 x G_{\mu\nu}^a F_{\mu\nu}^a$$
$$- \lambda_3 \frac{m^2}{16} \int \mathrm{d}^4 x \left(\overline{B}_{\mu\nu}^a - B_{\mu\nu}^a\right) G_{\mu\nu}^a$$
$$+ \gamma^2 \int \mathrm{d}^4 x \left(-g f^{abc} D_\mu^{bd} c^d \phi_\mu^{ac} + g f^{abc} A_\mu^b \omega_\mu^{ac}\right.$$
$$\left. + g f^{abc} D_\mu^{bd} c^d \overline{\phi}_\mu^{ac}\right)\,, \tag{122}$$

or

$$s'S_{\text{physical}} = \gamma^2 \int d^4x \left( -gf^{abc}D_\mu^{bd}c^d\phi_\mu^{ac} + gf^{abc}A_\mu^b\omega_\mu^{ac} \right.$$
$$\left. + gf^{abc}D_\mu^{bd}c^d\overline{\phi}_\mu^{ac} \right) . \tag{123}$$

Irrespective of the choice of BRST transformation $s$ or $s'$, the physical Gribov–Zwanziger action is not BRST invariant anymore. However, repeating the same argument given in the previous subsection, it turns out that the breaking terms can be treated consistently at the quantum level, leading to a renormalized broken Slavnov–Taylor identity. Furthermore, in most cases, everything will go as if the theory obeys an unbroken Slavnov–Taylor identity, because the breaking terms are in fact harmless.

# 9 Conclusions

In this paper, we have shown that the nonlocal gauge invariant operator $\text{Tr} \int d^4x F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ can be coupled to the Gribov–Zwanziger action in a localized form. By embedding this model into a larger class of models with local sources, we established a comprehensive set of Ward identities, which were sufficient to prove the renormalizability to all orders of perturbation theory. Specializing thus to a particular values of the local sources, we conclude that we have constructed a renormalizable action (103) that allows us to study the gauge invariant mass $m$ in combination with the restriction to the first Gribov horizon, obtained when the effective action is minimized with respect to the Gribov mass parameter $\gamma^2$. This restriction gives a first source of nonperturbative effects in gauge theories, as explained in the introduction: the gluon/ghost propagator gets infrared suppressed/enhanced, while the Gribov mass $\gamma$ is fixed in terms of the QCD scale $\Lambda_{\text{QCD}}$ by means of the requirement that the effective action is minimized with respect to it.

We also paid attention to the breaking of the BRST invariance when the sources are set equal to their physical values. We have elaborated on the fact that in most cases, the breaking terms in the Slavnov–Taylor identity are harmless, since they usually induce zero contributions to the identities between Green functions.

In the main body of this paper, we have extensively used and studied the BRST invariance in relation to the renormalizability. However, there is another major reason why the BRST symmetry is so important for perturbatively handled gauge theories. In a certain sense, BRST invariance is the quantum version of gauge invariance, and as such it should play a major role in reducing the number of relevant (physical) degrees of freedom, at least at the perturbative level. It is well known that a BRST symmetry with corresponding nilpotent charge is a very powerful tool in establishing the unitarity of gauge theories at the quantum level once a gauge has been chosen, see e.g. [38, 69–72].

When we discard the Gribov restriction, we retrieve the gauge model studied in [46, 63, 72], enjoying the BRST symmetry with nilpotent generator (120). Therefore, hope

existed that the model might be unitary, i.e. that one would be able to define a physical subspace $\mathcal{H}_{\text{phys}}$ of the total Hilbert state space $\mathcal{H}$, such that $\mathcal{H}_{\text{phys}}$ is endowed with a positive norm. This optimism turned out to be flawed, as it was shown in [72] that the massive gauge model is not unitary.

In addition to this, the restriction to the Gribov horizon only makes things worse. First of all, we have lost the (nilpotent) BRST symmetry, so any potential discussion of the unitarity cannot be based on BRST related tools. Further, we already mentioned in the introduction that the Gribov restriction gives rise to an infrared suppressed gluon propagator, and this suppression is so that the gluon propagator shows a violation of spectral positivity. Hence, the gluon is not expected to represent a physical particle, implying that we should certainly not expect unitarity from the Gribov–Zwanziger action when the gluons are treated as physical particles. We should rather expect the opposite. A hint that the Gribov restriction destabilizes the gluon is also given when we take a look at the tree level propagator, which in our conventions is given by [52]

$$\left\langle A_\mu^a A_\nu^b \right\rangle_p \equiv \delta^{ab} \frac{\mathcal{D}(p^2)}{p^2} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right),$$

with the gluon form factor

$$\mathcal{D}(p^2) = \frac{p^4}{p^4 + 2g^2N\gamma^4} , \tag{124}$$

which can also be written in a "standard" propagator form

$$\mathcal{D}(p^2) = \frac{1}{2}\frac{p^2}{p^2 + i\sqrt{2g^2N}\gamma^2} + \frac{1}{2}\frac{p^2}{p^2 - i\sqrt{2g^2N}\gamma^2} , \tag{125}$$

i.e. as the sum of two propagators with *imaginary* masses squared.

A similar reasoning can be applied when we do not implement the Gribov restriction. As we already outlined in [72], we can see the massless version of our model (112) as an alternative to ordinary Yang–Mills theory at high energies, based on their equivalence in the perturbative region [63]. The benefit of using our gauge model is that it is possible to couple mass terms $\propto m$ to it without jeopardizing the renormalizability. Then one can start looking for a sensible gap equation in order to generate a nonperturbative mass scale $m$. Stated otherwise, we could start looking for a gauge invariant dynamical mass generation mechanism. The generation of such a mass parameter would break the unitarity at the level of the gluons, but we must recall that unitarity is only a prerequisite for the *physical* degrees of freedom. We can depart our research from the massless (unitary) theory [72], but we are no longer interested in describing the perturbative asymptotic high energy regime of QCD, but instead we are entering a phenomenologically interesting region where e.g. the gluons already lose their physical meaning as observables. In this energy region, the gluons should be rather seen as a kind of quasi particles with a finite lifetime, thus not entering the asymptotic physical spectrum, which we do not know how to describe. We can continue to use the gluon propagator etc., albeit

the versions corrected by nonperturbative effects, like an effective gluon mass and/or Gribov restriction. At first instance, we can concentrate on the case with only the mass $m$ to be fixed, but at a later stage, we can also study the influence of the restriction to the Gribov region $\Omega$, since we have just proven the renormalizability of this action to all orders.

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